

## Event-B to lambdapi

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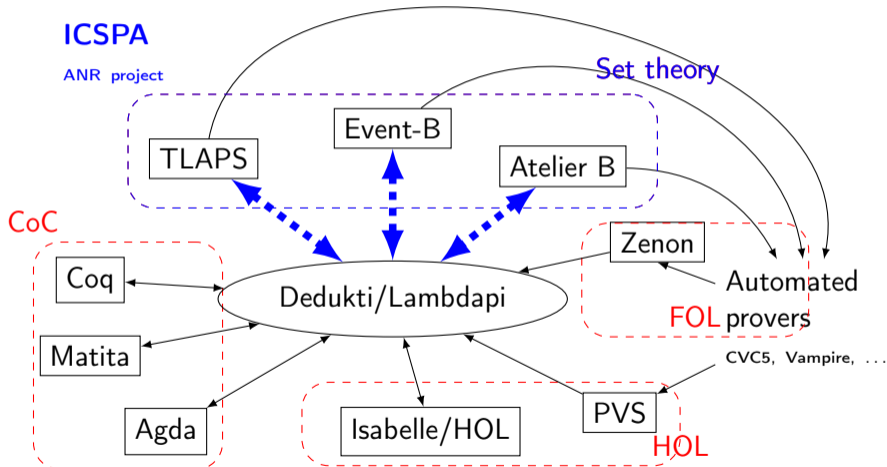
INP - IRIT  
Université de Toulouse  
Équipe ACADIE

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# Outline

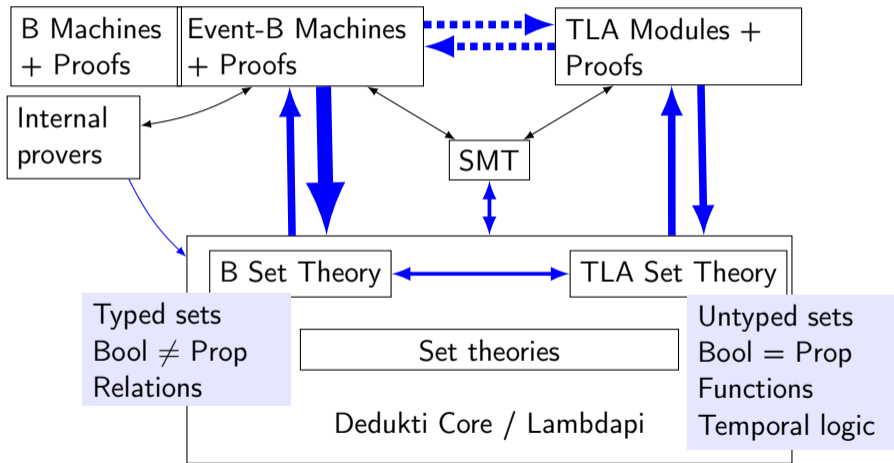
# ICSPA project

# Formal methods - Interoperability



- ICSPA Partners**
- SAMOVAR
  - INRIA Nancy
  - INRIA Paris-Saclay
  - IRIT
  - LIRMM
  - CLEARSY

## Formal methods based on set theories



## Mathematical constructs of Event-B

## B Mathematical Theory

The mathematical theory of Event-B, First Order Classical Predicate Calculus extended with Set Theory, is defined in several steps :

- Proposition language
- Predicate language
- Typed-set theory
- Arithmetic.

We will show the methodology with the construction of the propositional language and give some details on the typed-set theory.

## Proposition Language

### Basic constructs

1.  $\wedge, \Rightarrow, \neg$

→ Axiomatic theory

2. Constant  $\perp$  + more practical expression of rules.

Strategy → Semi-decision algorithm

	Antecedents	Consequent
R1	$H \vdash P$ $H \vdash Q$	$H \vdash P \wedge Q$
R2	$H \vdash P \wedge Q$	$H \vdash P$
R3	$H \vdash P \wedge Q$	$H \vdash Q$
R4	$H, P \vdash Q$	$H \vdash P \Rightarrow Q$
R5	$H \vdash P \Rightarrow Q$	$H, P \vdash Q$
R6	$H, \neg Q \vdash P$ $H, \neg Q \vdash \neg P$	$H \vdash Q$
R7	$H, Q \vdash P$ $H, Q \vdash \neg P$	$H \vdash \neg Q$



## Proposition Language

### Basic constructs

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Strategy →Semi-decision algorithm

	Antecedents	Consequent
INI	$H \vdash \neg R \Rightarrow \perp$	$H \vdash R$
AXM		$H, P, \neg P \vdash R$
AND1	$H \vdash \neg Q \Rightarrow R$ $H \vdash \neg P \Rightarrow R$	$H \vdash \neg(P \wedge Q) \Rightarrow R$
AND2	$H \vdash P \Rightarrow (Q \Rightarrow R)$	$H \vdash (P \wedge Q) \Rightarrow R$
IMP1	$H \vdash P \Rightarrow (\neg Q \Rightarrow R)$	$H \vdash \neg(P \Rightarrow Q) \Rightarrow R$
IMP2	$H \vdash Q \Rightarrow R$ $H \vdash \neg P \Rightarrow R$	$H \vdash (P \Rightarrow Q) \Rightarrow R$
NEG	$H \vdash P \Rightarrow R$	$H \vdash \neg\neg P \Rightarrow R$
DED	$H, P \vdash R$	$H \vdash P \Rightarrow R$

## Proposition Language

### Basic constructs

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Strategy →Semi-decision algorithm

	Antecedents	Consequent
INI	$H \vdash \neg R \Rightarrow \perp$	$H \vdash R$
AXM		$H, P, \neg P \vdash R$
AND1	$H \vdash \neg Q \Rightarrow R$ $H \vdash \neg P \Rightarrow R$	$H \vdash \neg(P \wedge Q) \Rightarrow R$
AND2	$H \vdash P \Rightarrow (Q \Rightarrow R)$	$H \vdash (P \wedge Q) \Rightarrow R$
IMP1	$H \vdash P \Rightarrow (\neg Q \Rightarrow R)$	$H \vdash \neg(P \Rightarrow Q) \Rightarrow R$
IMP2	$H \vdash Q \Rightarrow R$ $H \vdash \neg P \Rightarrow R$	$H \vdash (P \Rightarrow Q) \Rightarrow R$
NEG	$H \vdash P \Rightarrow R$	$H \vdash \neg\neg P \Rightarrow R$
DED	$H, P \vdash R$	$H \vdash P \Rightarrow R$

Order of rules : AXM, IMP1, IMP2, AND1, AND2, NEG

Proof procedure : INI ; (RULES\* ;DED)\*

## Propositional calculus

### Derived constructs

$\vee$ ,  $\Leftrightarrow$  and  $\top$ , defined as rewriting of basic constructs.

Predicate	Definition
$\top$	$\neg \perp$
$P \vee Q$	$\neg P \Rightarrow Q$
$P \Leftrightarrow Q$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

### Derived rules

Proved with previous rules.

	Antecedents	Consequent
OR1	$H \vdash \neg P \Rightarrow (\neg Q \Rightarrow R)$	$H \vdash \neg(P \vee Q) \Rightarrow R$
OR2	$H \vdash Q \Rightarrow R$ $H \vdash P \Rightarrow R$	$H \vdash (P \vee Q) \Rightarrow R$
EQV1	$H \vdash P \Rightarrow (\neg Q \Rightarrow R)$ $H \vdash \neg P \Rightarrow (Q \Rightarrow R)$	$H \vdash (\neg P \Leftrightarrow Q) \Rightarrow R$
EQV2	$H \vdash P \Rightarrow (Q \Rightarrow R)$ $H \vdash \neg P \Rightarrow (\neg Q \Rightarrow R)$	$H \vdash (P \Leftrightarrow Q) \Rightarrow R$

## Derived rules

With these rules, we can prove some classical results : commutativity, associativity, distributivity, law of excluded middle, idempotence, absorption, de Morgan laws, contraposition, double negation, transitivity, monotony, equivalence, like :

For  $P$  and  $Q$  predicates :

$P \vee \neg P$	Law of excluded middle
$P \Leftrightarrow \neg\neg P$	Double negation
$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$	de Morgan laws
$P \vee P \Leftrightarrow P$ $P \wedge P \Leftrightarrow P$	Idempotence
$(P \vee Q) \wedge P \Leftrightarrow P$ $(P \wedge Q) \vee P \Leftrightarrow P$	Absorption

## Equivalence rewriting

For  $P, Q, R$  predicates, such as  $P \Leftrightarrow Q$  :

$$\begin{array}{l} (P \wedge R) \Rightarrow (Q \wedge R) \\ (P \vee R) \Rightarrow (Q \vee R) \\ (R \Rightarrow P) \Rightarrow (R \Rightarrow Q) \\ (Q \Rightarrow R) \Rightarrow (P \Rightarrow R) \\ \neg P \Rightarrow \neg Q \end{array}$$

« *The last series of properties shows that when two predicates have been proved to be equivalent then replacing one by the other in any predicate preserves equivalence (this can be proved by induction on the syntactic structure of the predicate notation). In other words, once proved, an equivalence assertion can be used operationally as if it were a rewriting rule.* »<sup>1</sup>

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1. J.-R. Abrial. The B-Book, assigning programs to meaning. Cambridge University Press, 1996.

# First order predicate calculus

## Predicates language

Following the same methodology, we define :

- Variables, expressions, substitutions,
- Basic predicate universal quantifier  $\forall$ ,
- Derived predicate universal quantifier  $\exists$
- Definition of equality.

## First order predicate calculus

<i>predicate</i>	<i>:=</i>	$\perp$ $\top$ <i>predicate</i> $\wedge$ <i>predicate</i> <i>predicate</i> $\vee$ <i>predicate</i> <i>predicate</i> $\Rightarrow$ <i>predicate</i> <i>predicate</i> $\Leftrightarrow$ <i>predicate</i> $\forall$ <i>varList</i> . <i>predicate</i> $\exists$ <i>varList</i> . <i>predicate</i> [ <i>varList</i> := <i>expList</i> ] <i>predicate</i> <i>expression</i> = <i>expression</i>
<i>expression</i>	<i>:=</i>	<i>variable</i> [ <i>varList</i> := <i>expList</i> ] <i>expression</i> <i>expression</i> $\rightarrow$ <i>expression</i>
<i>variable</i>	<i>:=</i>	<i>identifier</i>

## Event-B Set theory

We extend the theory with the syntactic category *set* and the *membership* predicate :  $E \in s$ ,  $E$  expression and  $s$  set.

Some rules :

$$E \in Pow(S) \Rightarrow \forall x. x \in E \Rightarrow x \subset S$$

$$S \subset T \Rightarrow S \in \mathbb{P}(T)$$

$$E \in S \cap T \Rightarrow E \in S \wedge E \in T$$

This completes the syntax :

<i>predicate</i>	$:=$	$\dots$ $expression \in expression$
<i>expression</i>	$:=$	$\dots$ <i>set</i>
$\dots$		
<i>set</i>	$:=$	$set \times set$ $\mathbb{P}(set)$ $\{varList.predicate   expression\}$ <i>variable</i>



## Event-B Type theory

Any predicate will be type-checked before being proved. A type denotes the set of values an expression can take.

### Event-B types :

$T ::=$	BOOL		$\mathbb{Z}$	built-in boolean and integer types
		$S$		carrier set $S$ provided by user
		$\mathbb{P} T$		power set of a type
		$T \times T$		cartesian product of types

## Embedding Event-B in lambdapi

# Lambdapi

« *Lambdapi is an interactive proof system featuring **dependent types** like in Martin-Löf's type theory, but allowing to define objects and types using **oriented equations**, aka **rewriting rules**, and reason modulo those equations.* »<sup>2</sup>

## $\lambda\Pi$ terms

$t, t' ::=$	$V$	variable
	TYPE	sort for types
	$\Pi (V : t), t'$	dependent product type
	$\lambda (V : t), t'$	abstraction
	$t t'$	application
	$t \rightarrow t'$	abbreviation for $\Pi (V : t), t'$ when $V \notin t'$

## Rules

$r ::= t \leftrightarrow t'$       reasoning modulo rewriting rules

2. <https://lambdapi.readthedocs.io/en/latest/about.html>

## First order logic<sup>3</sup>

« *Lambdapi* is a logical framework, that is, it does not come with a pre-defined logic. Instead, one has to start defining its own logic. »

### Propositional logic

```
constant symbol Prop : TYPE;  
// Associates a type of a proof to a proposition  
injective symbol  $\pi$  : Prop  $\rightarrow$  TYPE;
```

### Types of datatypes

```
constant symbol Set : TYPE;  
// Associates a type to a datatype  
injective symbol  $\tau$  : Set  $\rightarrow$  TYPE;
```

---

3. Standard library : <https://github.com/Deducteam/lambdapi-stdlib>

## First order logic

« *Lambdapi* is a logical framework, that is, it does not come with a pre-defined logic. Instead, one has to start defining its own logic. »

### Conjunction

```
constant symbol  $\wedge$  : Prop  $\rightarrow$  Prop  $\rightarrow$  Prop;
notation  $\wedge$  infix left 7;
constant symbol  $\wedge_i$  p q:  $\pi$  p  $\rightarrow$   $\pi$  q  $\rightarrow$   $\pi$  (p  $\wedge$  q);
symbol  $\wedge_{e1}$  p q :  $\pi$  (p  $\wedge$  q)  $\rightarrow$   $\pi$  p;
symbol  $\wedge_{e2}$  p q :  $\pi$  (p  $\wedge$  q)  $\rightarrow$   $\pi$  q;
```

### Implication (Coq style)

```
constant symbol  $\Rightarrow$  : Prop  $\rightarrow$  Prop  $\rightarrow$  Prop;
notation  $\Rightarrow$  infix right 5;
rule  $\pi$  ($p  $\Rightarrow$  $q)  $\hookrightarrow$   $\pi$  $p  $\rightarrow$   $\pi$  $q;
```

Related sequents  
for conjunction

$$\frac{\Gamma \vdash p \quad \Gamma \vdash q}{\Gamma \vdash p \wedge q} (\wedge_i)$$

$$\frac{\Gamma \vdash p \wedge q}{\Gamma \vdash p} (\wedge_{e1})$$

$$\frac{\Gamma \vdash p \wedge q}{\Gamma \vdash q} (\wedge_{e2})$$

## Event-B set theory

### Event-B types :

$S ::= \sigma\mathbb{P} S$	power set
$S \sigma\times S$	cartesian product
$\sigma\text{BOOL} \mid \sigma\mathbb{Z}$	built-in boolean and integer types
$\sigma S$	for each user declared set $S$

### In `lambdapi` :

```

injective symbol  $\sigma\mathbb{P}$ : Set  $\rightarrow$  Set; // power set
injective symbol  $\sigma\times$ : Set  $\rightarrow$  Set  $\rightarrow$  Set; // cartesian product
notation  $\sigma\times$  infix left 24;
constant symbol  $\sigma\text{BOOL}$ : Set; // pre-defined boolean set
constant symbol  $\sigma\mathbb{Z}$ : Set; // pre-defined integer set
constant symbol  $\sigma S$ : Set; // user declared set S
  
```

## Set operators

Classical set operators of Event-B derive from membership operator :

```

symbol  $\in$  [T:Set] :  $\tau$  T  $\rightarrow$   $\tau$  ( $\sigma\mathbb{P}$  T)  $\rightarrow$  Prop;
rule  $\_ \in \emptyset \leftrightarrow \perp$ ;
rule  $\$x \in \$s1 \cap \$s2 \leftrightarrow \$x \in \$s1 \wedge \$x \in \$s2$ ;
rule  $\$e \in \mathbb{P} \$S \leftrightarrow \$e \subseteq \$S$ ;
rule  $\$s1 \subseteq \$s2 \leftrightarrow \forall x, x \in \$s1 \Rightarrow x \in \$s2$ ;

```

### Generic maximal set BIG

```

constant symbol BIG [T:Set]:  $\tau$  ( $\sigma\mathbb{P}$  T); // set of all elements of type  $\tau$  T
rule  $\$x \in \text{BIG} \leftrightarrow \top$ ; // BIG is maximal: contains all elements of type  $\tau$  T
rule  $\mathbb{P} \text{BIG} \leftrightarrow \text{BIG}$ ; // power set of BIG is a maximal set
rule  $\text{BIG} \times \text{BIG} \leftrightarrow \text{BIG}$ ; // cartesian product of two maximal sets is maximal

```

## Critical pairs

- $P \wedge \top \Rightarrow P$
- $P \vee \top \Rightarrow \top$
- In Rodin, the rule type rewrites do some automatic rewriting :  $x \in S$  if  $S$  is maximal, then  $x \in S \Leftrightarrow \top$ . The choice of BIG and its rules is a solution to express some of these rules, but this is also a source of conflicts.

### Example :

- $x \in \mathbb{P}(\mathit{BIG})$
- $x \in \mathbb{P}(\mathit{BIG})$



## Critical pairs

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### Example :

- $x \in \mathbb{P}(BIG) \leftrightarrow x \subseteq BIG$
- $x \in \mathbb{P}(BIG)$

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### Example :

- $x \in \mathbb{P}(BIG) \leftrightarrow x \subseteq BIG \leftrightarrow \forall u. u \in x \Rightarrow u \in BIG$

As  $u \in BIG \leftrightarrow \top$ , we have  $\forall u. u \in x \Rightarrow \top$

- $x \in \mathbb{P}(BIG)$

## Critical pairs

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### Example :

- $x \in \mathbb{P}(BIG) \leftrightarrow x \subseteq BIG \leftrightarrow \forall u. u \in x \Rightarrow u \in BIG \leftrightarrow \forall u. u \in x \Rightarrow T$   
As  $u \in BIG \leftrightarrow T$ , we have  $\forall u. u \in x \Rightarrow T$
- $x \in \mathbb{P}(BIG)$

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### Example :

- $x \in \mathbb{P}(\mathbf{BIG}) \leftrightarrow x \subseteq \mathbf{BIG} \leftrightarrow \forall u. u \in x \Rightarrow u \in \mathbf{BIG} \leftrightarrow \forall u. u \in x \Rightarrow T$   
As  $u \in \mathbf{BIG} \leftrightarrow T$ , we have  $\forall u. u \in x \Rightarrow T$
- $x \in \mathbb{P}(\mathbf{BIG}) \leftrightarrow x \in \mathbf{BIG} \leftrightarrow T$

## Relational operators

```
symbol rel (T1 T2: Set) =  $\tau$  ( $\sigma\mathbb{P}$  (T1  $\sigma\times$  T2));  
injective symbol  $\mapsto$  [T1:Set] [T2:Set] (x: $\tau$  T1) (y: $\tau$  T2) :  $\tau$  (T1  $\sigma\times$  T2);  
symbol  $\leftrightarrow$  [T1:Set] [T2:Set] (A: $\tau$  ( $\sigma\mathbb{P}$  T1)) (B:  $\tau$  ( $\sigma\mathbb{P}$  T2)):  
   $\tau$  ( $\sigma\mathbb{P}$  ( $\sigma\mathbb{P}$  (T1  $\sigma\times$  T2))) =  $\mathbb{P}$  (A  $\times$  B); notation  $\leftrightarrow$  infix 11;  
  
constant symbol dom [T1:Set] [T2:Set] : rel T1 T2  $\rightarrow$   $\tau$  ( $\sigma\mathbb{P}$  T1);  
notation dom prefix 30;  
rule  $\$x \in \text{dom}(\$r) \leftrightarrow \exists y, \$x \mapsto y \in \$r$ ;
```

## Proofs in Rodin



# Rodin : Rigorous Open Development Environment for Complex Systems

The Rodin Platform is an Eclipse-based IDE for Event-B that provides effective support for refinement and mathematical proof. The platform is open source, contributes to the Eclipse framework and is further extendable with plugins. <sup>4</sup>

The mathematical proofs are shown as **proof trees**.  
Statements are declared in Contexts and the proof trees are built automatically and/or guided by the user.

We will present these notions using the example of Cantor's Theorem.

---

4. <https://www.event-b.org/platform.html>

## Context - Cantor's theorem

### In Rodin

```
cantor ×  
context cantor  
  
sets S  
  
axioms  
  theorem @th ¬(∃f · f ∈ S → P(S))  
end
```

### In Lambdapi

```
constant symbol σS: Set;  
symbol S: τ (σP σS) := BIG;
```

**S** is the embedding in lambdapi of the set **S** defined in the Rodin-context.

## Cantor's theorem

## In Rodin

```

cantor ×
context cantor

sets S

axioms
  theorem @th ¬(∃f·f∈S→P(S))
end

```

## In Lambdapi

$f$  is a total surjection from  $S$  to  $\mathbb{P}((S))$ ,  
that is  $f \in \mathbb{P}(S \times \mathbb{P}((S)))$  and a total  
function and a surjection.

```

constant symbol σS: Set;
symbol S: τ (σP σS) = BIG;
symbol th: π(¬((∃ (f: τ(σP (σS σ × (σP σS))))), f ∈ (S → (P S)))) :=
... end;

```

# Building Cantor's proof with Rodin

The screenshot displays the Rodin Platform interface for the file `rodin-workspace - TEST_ICSPA/cantor.bps`. The interface is divided into several panels:

- Proof Tree (Left):** A hierarchical view of the proof goals. The current goal is  $\exists \text{hyp } (\exists T.T = \{x \mid \forall U.x \mapsto U \text{ef} \Rightarrow x \in U\})$ . A red arrow points from this goal to the `Remove membership` button in the central panel.
- Central Panel:** Shows the current goal `th/THM` and its hypotheses: `f ∈ S → P(S)` and `T = {x. ∀U. x ↦ U ef ⇒ x ∈ U | x}`. A red arrow points to the `Remove membership` button, which is highlighted with a red circle.
- Event-B Explorer (Right):** A tree view of the project structure, including `TEST_ICSPA`, `cantor`, `Carrier Sets`, `Constants`, `Axioms`, and `Proof Obligations`.
- Proof Control (Bottom):** A toolbar with various icons for proof control, including `app`, `pp`, `dc`, `ah`, `ae`, `p0`, `p1`, `ml`, and `SMT`.

A status bar at the bottom left indicates: **Tactic applied successfully**.

# Building Cantor's proof with Rodin

The screenshot shows the Rodin Platform interface for a proof workspace named 'TEST\_ICSPA/cantor.bps'. The interface is divided into several panels:

- Proof Tree (Left):** A hierarchical view of the proof. The goal 'remove ∈ in feS → P(S)' is selected and highlighted in blue. Below it, the hypothesis 'eh with T={x·VU·x → Uef⇒→x#U}' is visible.
- Goal Panel (Center):** Displays the current goal 'remove ∈ in feS → P(S)' and its hypothesis 'T={x·VU·x → Uef⇒→x#U | x}'. A red circle highlights the hypothesis.
- Event-B Explorer (Right):** A tree view of the project structure, showing 'TEST\_ICSPA' and its sub-elements: 'cantor', 'Carrier Sets', 'Constants', 'Axioms', and 'Proof Obligations'.
- Rule Details (Bottom Right):** A panel showing the details of the selected rule. It includes the rule name 'remove ∈ in feS → P(S)', the antecedent 'Antecedent1', and the rewrite rule:
 
$$\text{feS} \rightarrow \text{P(S)}$$

$$\vdash \text{feS} \leftrightarrow \text{P(S)}$$

$$\text{dom}(f)=S$$
 The selected rule is also shown as:
 
$$\text{Select: } \text{feS} \leftrightarrow \text{P(S)}$$

$$\text{dom}(f)=S$$
- Proof Control (Bottom):** A toolbar with various proof tactics like 'app', 'pp', 'K', 'dc', 'ah', 'ae', 'p0', 'p1', 'mi', and 'SMT'.

A red arrow points from the selected goal in the Proof Tree to the Rule Details panel. At the bottom left, the text 'Select a new proof node' is displayed.

## Proof tree

Proof Tree X

- v ✓ remove  $\neg$  in goal
  - v ✓  $\forall$  goal (frees  $f$ )
    - v ✓ ct goal
      - v ✓ remove  $\in$  in  $f \in S \rightarrow P(S)$ 
        - v ✓ remove  $\in$  in  $f \in S \leftrightarrow P(S)$ 
          - v ✓ remove  $\in$  in  $f \in S \leftrightarrow P(S)$ 
            - v ✓ type rewrites
              - v ✓ rewrites set equality in hyp ( $\text{ran}(f)=P(S)$ )
                - v ✓ type rewrites
                  - v ✓ remove  $\subseteq$  in  $P(S) \subseteq \text{ran}(f)$ 
                    - v ✓ type rewrites
                      - v ✓ simplification rewrites
                        - v ✓  $\forall$  hyp (inst  $\{x \mid \forall U \cdot x \mapsto U \in f \Rightarrow \neg x \in U\}$ )
                          - ✓  $\top$  goal
                            - v ✓ remove  $\in$  in  $\{x \mid \forall U \cdot x \mapsto U \in f \Rightarrow \neg x \in U\} \in \text{ran}(f)$ 
                              - v ✓  $\exists$  hyp ( $\exists x \cdot x \mapsto \{x \mid \forall U \cdot x \mapsto U \in f \Rightarrow \neg x \in U\} \in f$ )
                                - v ✓ dc ( $x \in \{x \mid \forall U \cdot x \mapsto U \in f \Rightarrow \neg x \in U\}$ )
                                  - ✓  $\top$  goal

## Translation of the Rules from Event-B to lambdapi

| Rodin proof rule  | Lambdapi tactic                                    |
|---|--|
| $\frac{}{\Gamma, h : p \vdash p} \text{ (hyp)}$   | refine $h$   |
| $\frac{h : p, \Gamma \vdash q}{\Gamma \vdash p \Rightarrow q} \text{ (}\Rightarrow \text{ goal)}$                           | assume $h$   |
| $\frac{\Gamma, h : x_i \in T_i \vdash p}{\Gamma \vdash \forall x_1, \dots, x_n \cdot p} \text{ (}\forall \text{ goal)}$     | assume $x_1 \dots x_n$                             |
| $\frac{\Gamma \vdash p_1 \dots \Gamma \vdash p_n}{\Gamma \vdash p_1 \wedge \dots \wedge p_n} \text{ (}\wedge \text{ goal)}$ | apply $\wedge_i p_1$ ( apply $\wedge_i p_2$ (...)) |

## Rules from Event-B to lambdapi

### Rules defined as theorems

```

symbol Or2ImpGoal [P Q: Prop] :
  π (((¬ P) ⇒ Q) ⇒ (P ∨ Q)) ≡
begin
  assume P Q h;
  apply (λ h1 h2, ∨e P (¬ P)
        (P ∨ Q) h1 h2 (classic P))
    {assume hp; apply (∨i1 _ _ hp)}
    {assume hnp; apply (∨i2 _ _ (h hnp))}
end;

```

| Rodin proof rule  | Lambdapi tactic  |
|---|------------------|
| $\frac{\Gamma \vdash \neg p \Rightarrow q}{\Gamma \vdash p \vee q}$ | apply Or2ImpGoal |



# Conclusion

# Open topics

## Problems

- lot's of automatic rewriting rules in Event-B/Rodin
- Prop and Bool are different in Rodin, = and  $\Leftrightarrow$  can't be identify
- some operators, like  $\wedge$  or  $\vee$  are n-ary, difficult to express in lambdapi

## Open topics

### Problems

- lot's of automatic rewriting rules in Event-B/Rodin
- Prop and Bool are different in Rodin, = and  $\Leftrightarrow$  can't be identify
- some operators, like  $\wedge$  or  $\vee$  are n-ary, difficult to express in lambdapi

### Investigations

- Use Lambdapi rewriting rules , but too much rewriting rules leads to critical pairs (eg. BIG, former neg)
- Theorems, preprocessing and tactics (repeat, setoid rewrite,...) with synthesis of lambdapi proof term in Java.
- Integration of Coq<sup>5</sup> setoid rewrite in Lambdapi ?

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5. <https://coq.inria.fr/doc/V8.10.2/refman/addendum/generalized-rewriting.html>

## Generalized rewriting

### Rewriting rules

- equal by equal rewriting :  $((a = b) ==> f(a)) \Rightarrow f(b)$
- equivalent by equivalent rewriting :  $((P \Leftrightarrow Q) ==> f(P)) \Rightarrow f(Q)$
- $P \wedge \top \Leftrightarrow P$
- $P \wedge \dots Q \wedge P \wedge R \dots \Leftrightarrow P \wedge \dots Q \wedge R \dots \dots$

### In Lambdapi

Tactic `rewrite`<sup>6</sup> allows rewriting only for equality, not for equivalence.

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6. <https://lambdapi.readthedocs.io/en/latest/tactics.html>  
<https://inria.hal.science/inria-00258384>

Dedukti/Lambdapi is a logical framework based on  $\lambda\Pi$ -calculus modulo rewriting system, meant to allow interoperability between formal method systems.

We presented some steps of our translation of the first order logic and set theory of Event-B and its deduction rules in Lambdapi to translate a statement and a guided proof from Rodin in Lambdapi.

A first usecase has been a guided proof of Cantor's theorem in Event-B.

### Ongoing work

- Continue translation of deduction rules
- Deals with generalized rewriting
- Deals with internal and external automated provers
- Translate machines and events

Thanks for your attention