EuroProofNet WG1+2+4 meeting

Dependent Ghosts have a reflection for free

Théo Winterhalter

What's up with vectors?

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Inductive vec A : ℕ → Type :=
| vnil : vec A 0
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The problem is always in the n of `vec A n`...

Using ghost types…

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Inductive vec A : erased ℕ → Type :=
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but…

```
erased bool → bool
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only contains constant functions

…and ghost reflection

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A : Ghost     u, v : A     e : u = v
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Propositionally equal inhabitants of ghosts
are definitionally equal

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ok because `vec A (gS k +’ m) ≡ vec A (k +’ gS m)`

Wait, couldn't this just be (S)Prop?

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Inductive erased (A : Type) : Ghost :=
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Eliminator `reveal` cannot go to `Type`
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$$\frac{A : \mathtt{Ghost} \qquad u, v : A \qquad e : u = v}{u \equiv v}$$

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Inductive squash (A : Type) : Prop :=
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Not if we want to distinguish the two types

`vec A (hide 0)` and `vec A (gS n)` *(eg to build head and `tail` functions)*

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this is a problem though!

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Reveal proposition

$$\frac{\texttt{e : erased A} \qquad \texttt{f : A → Prop}}{\texttt{Reveal e f : Prop}}$$

```
Reveal (hide t) f ↔ f t
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Reveal proposition

$$\frac{e : \mathsf{erased}\ A \qquad f : A \to \mathsf{Prop}}{\mathsf{Reveal}\ e\ f : \mathsf{Prop}}$$

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Reveal (hide t) f ↔ f t
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We get a discriminator: `D (hide 0) ↔ ⊤` `D (gS n) ↔ ⊥`

```
D : erased ℕ → Prop
D n := Reveal n (λx. match x with 0 => ⊤ | _ => ⊥ end)
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How do we justify this?

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Ghost reflection

↓

$$\frac{A : \texttt{Ghost} \qquad e : u =_A v \qquad t : P\ u}{\texttt{cast}\ e\ P\ t : P\ v}$$

Ghost casts

How do we justify this?

$$\frac{\texttt{A : Ghost} \qquad \texttt{u, v : A} \qquad \texttt{e : u = v}}{\texttt{u} \equiv \texttt{v}}$$

Ghost reflection

translating *derivations*
replacing conversion rule by casts
like for ETT to ITT [Oury 2005 ; WST 2019]

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MLTT/CIC with (S)Prop

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taking inspiration from exceptional type theory
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interesting on its own!

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MLTT/CIC with (S)Prop

Erasure

Translation getting rid of all ghosts and proofs

$$[\lambda(x^{\mathbb{T}} : A).\ t]_\varepsilon := \lambda(x : [\![A]\!]_\varepsilon).\ [t]_\varepsilon$$

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$A : \mathrm{Type} \longrightarrow [\![A]\!]_\varepsilon : \mathrm{Type}$, $[\![A]\!]_\varnothing : [\![A]\!]_\varepsilon$

Example

Booleans

source

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Inductive bool :=
| true
| false
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parametricity in Prop [Keller Lasson 2012]

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Inductive boolP : bool• → Prop :=
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predicate guaranteeing no exceptions raised at top-level

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Free theorem:

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erased bool → bool
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only contains constant functions

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```
Inductive vec• (A : ty) :=
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| vec∅
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Inductive vecP (A : ty) (AP : El A → Prop) : ∀ n (nP : ℕP n), vec• A → Prop :=
| vnilP : vecP A AP 0• 0P vnil•
| vconsP a (aP : AP a) n nP v : vecP A AP n nP v → vecP A AP (S• n) (SP n nP) (vcons• a v)
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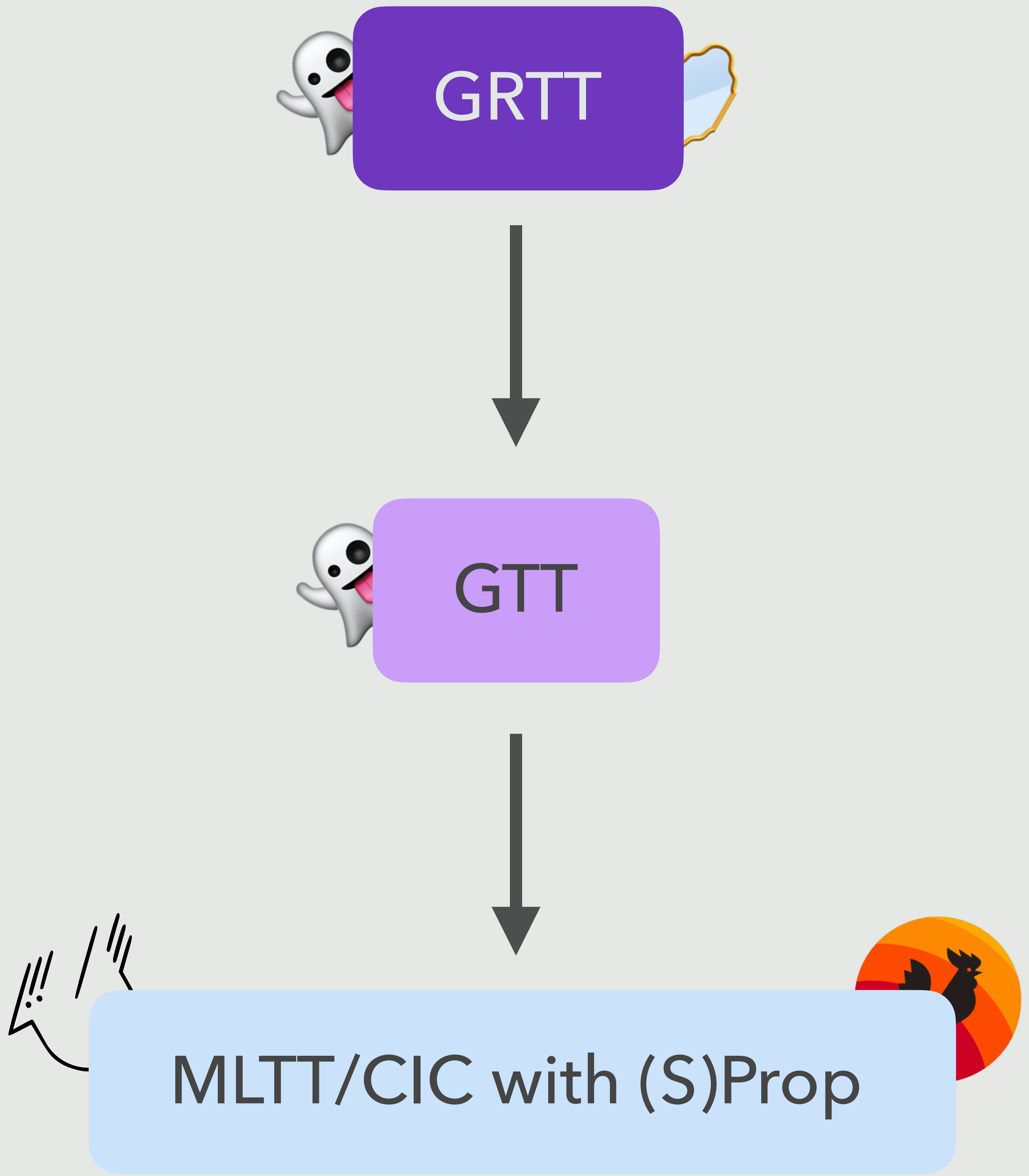
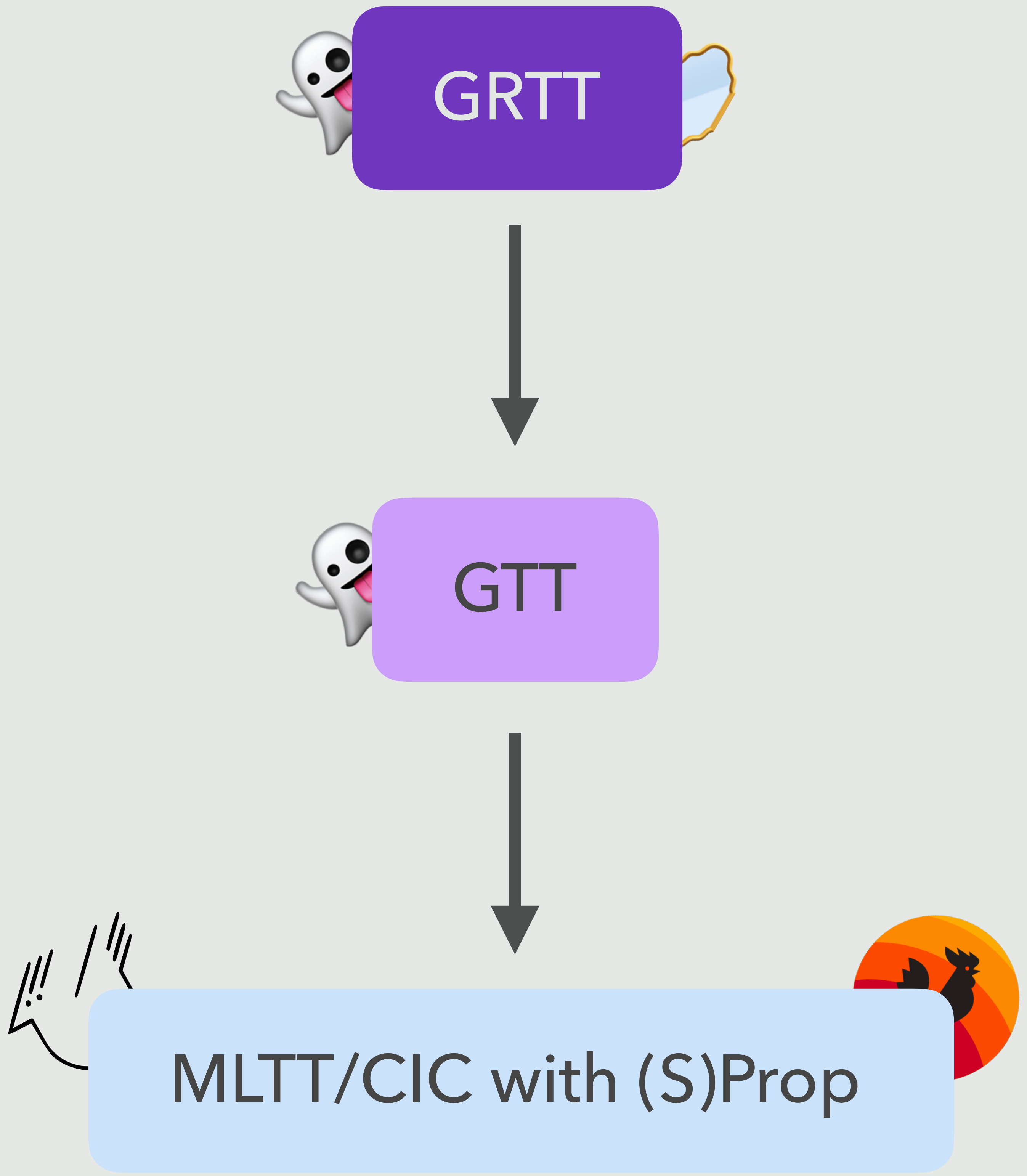
```

+ extra oddities in the paper


GRTT
GTT
MLTT/CIC with (S)Prop

Meta-theory

conservativity
consistency
type former discrimination
free theorems



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conservativity
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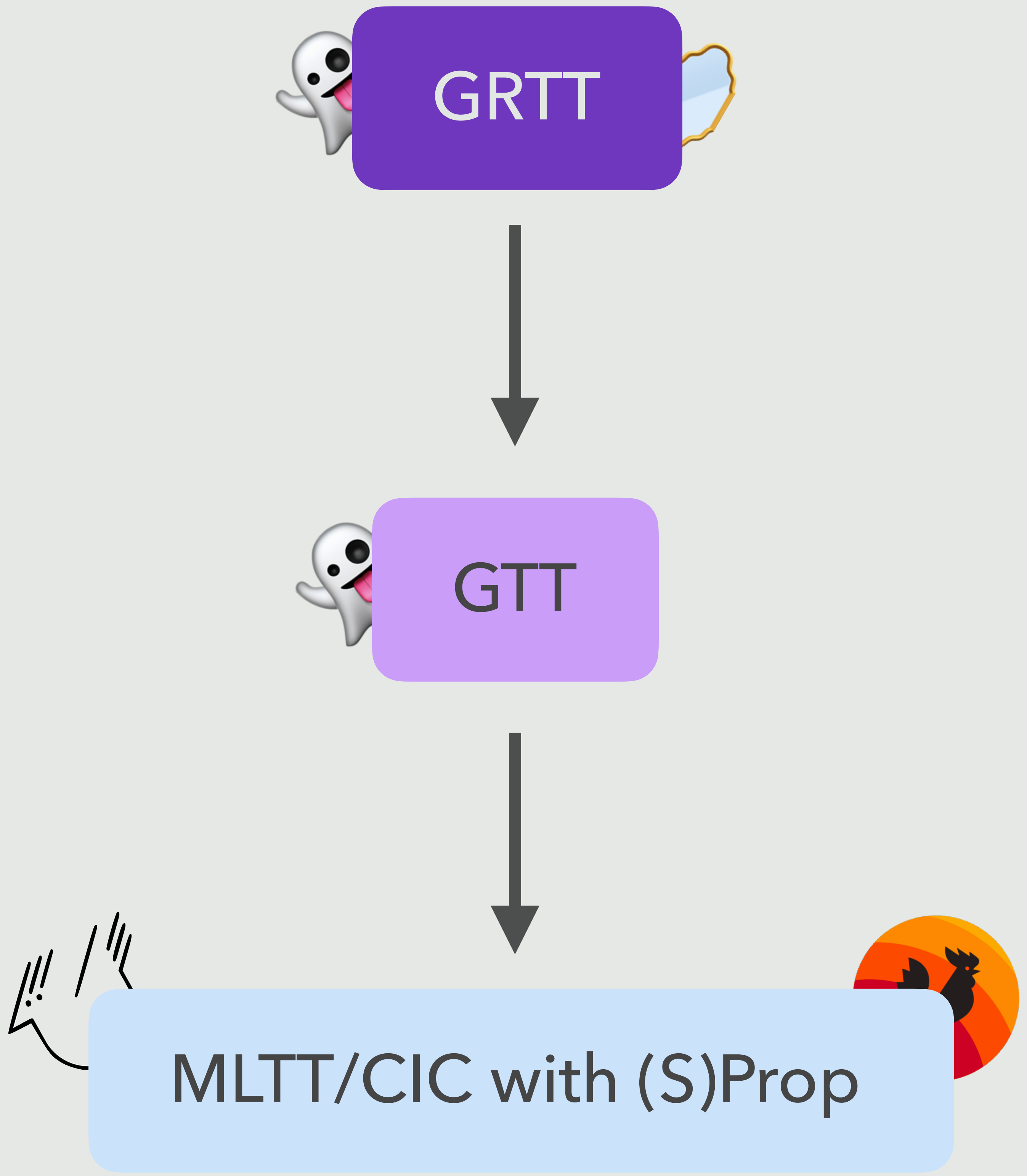

Perspectives

general inductives
subject reduction
termination
decidability (for GTT only)
meta-theory of F*

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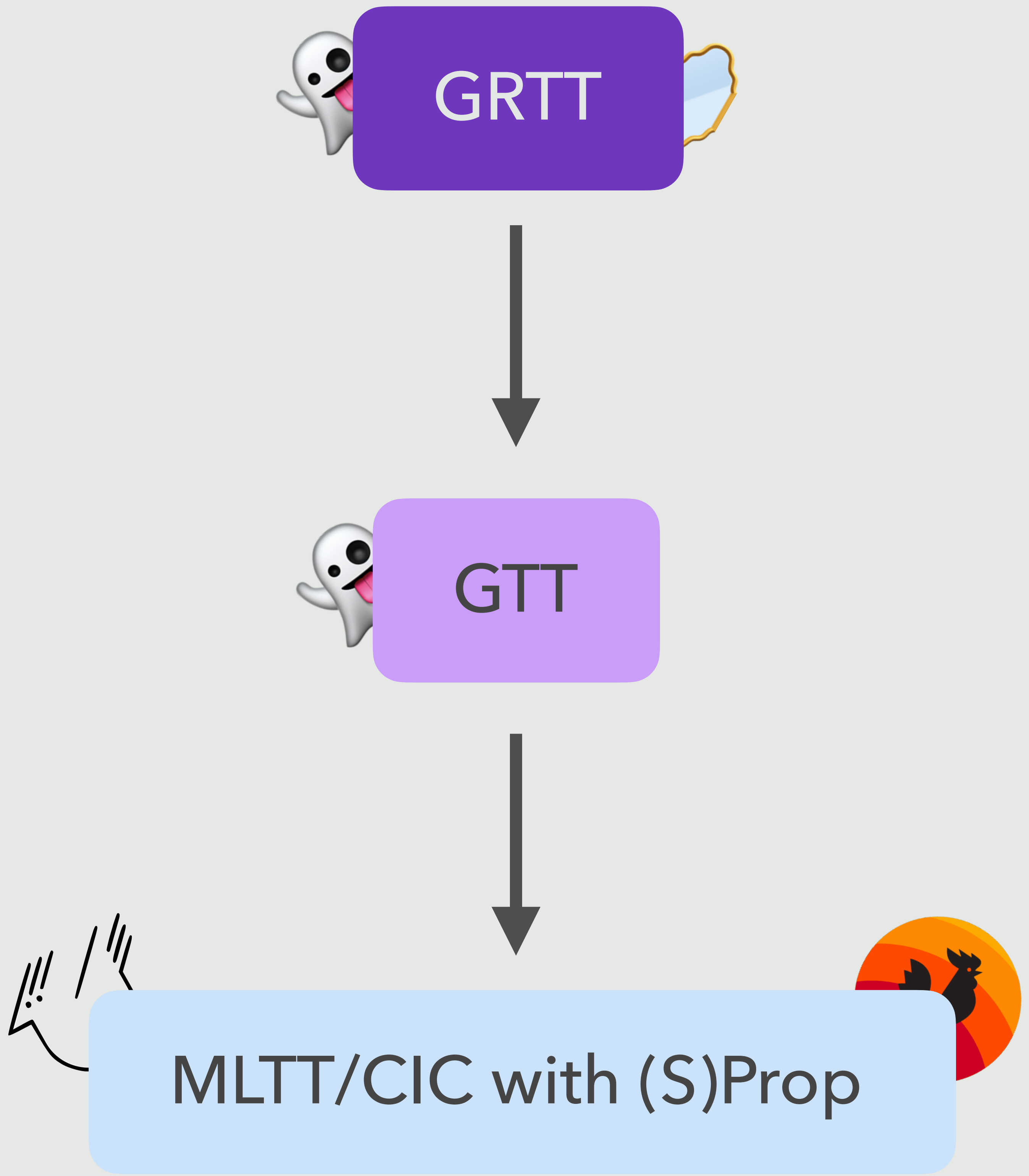

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ongoing work by **Ewen Broudin--Caradec**

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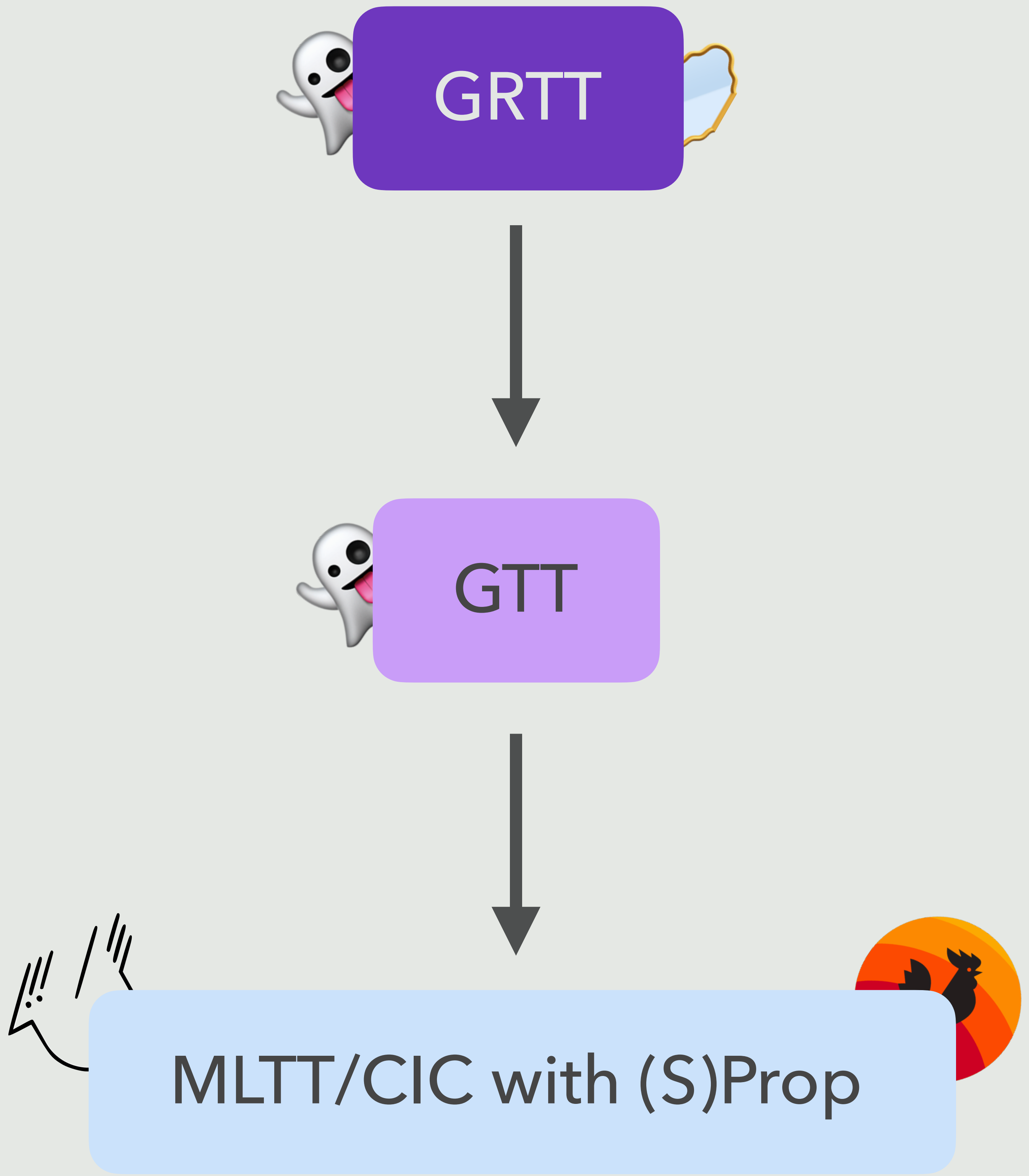

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ongoing work by **Ewen Broudin--Caradec**

some tricks in the formalisation to handle contexts of varying size

Autosubst 2 very useful but had to rewrite automation

/TheoWinterhalter/ghost-reflection