Non-trivial Multi-Modal Logics with Interactions

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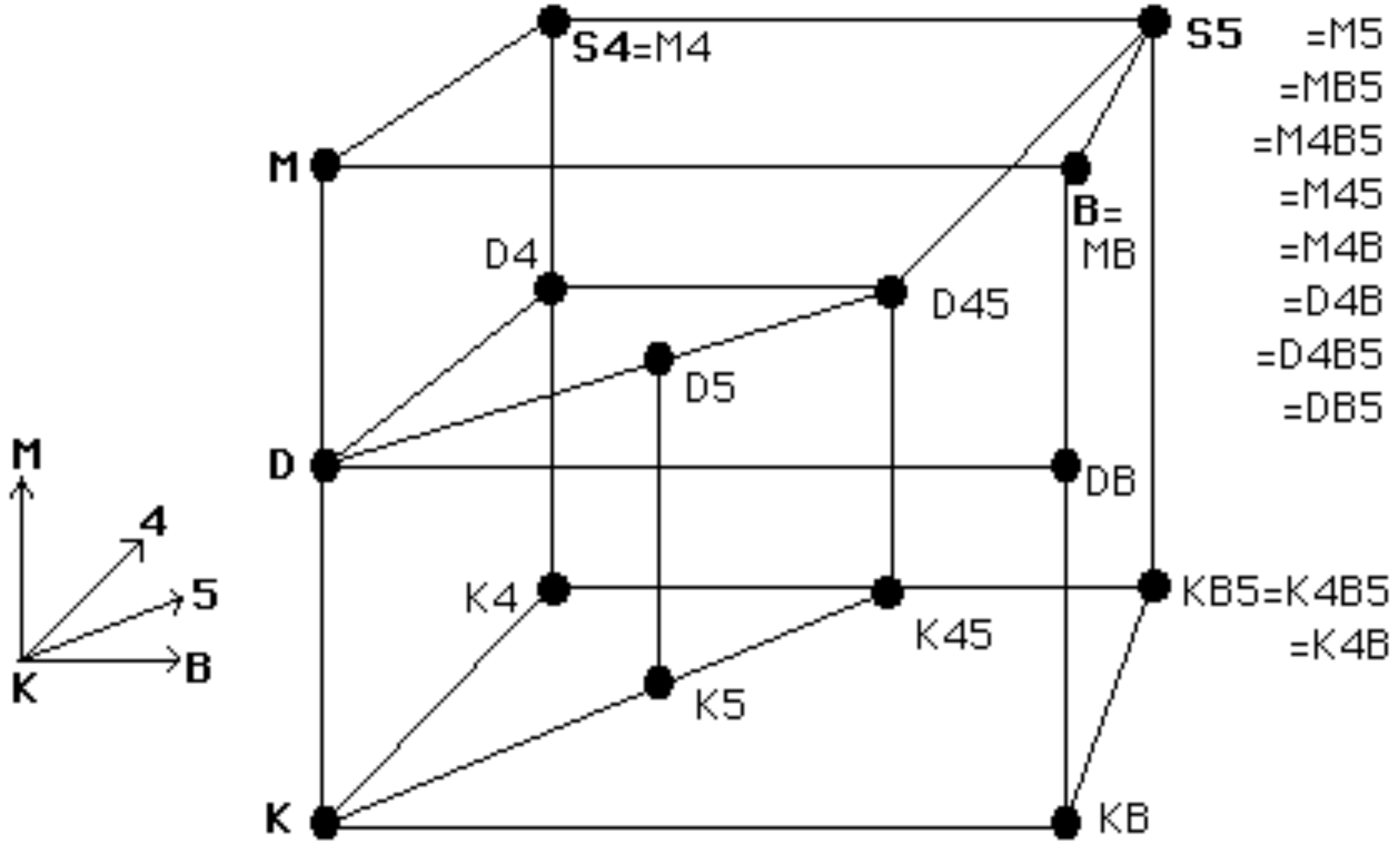
Characterisation of modal logics:

Non-trivial

Characterisation of modal logics:

• Axiom schemes	
(M)	$\Box A \supset A$
(4)	$\Box A \supset \Box\Box A$
(5)	$\Diamond A \supset \Box\Diamond A$
(B)	$A \supset \Box\Diamond A$

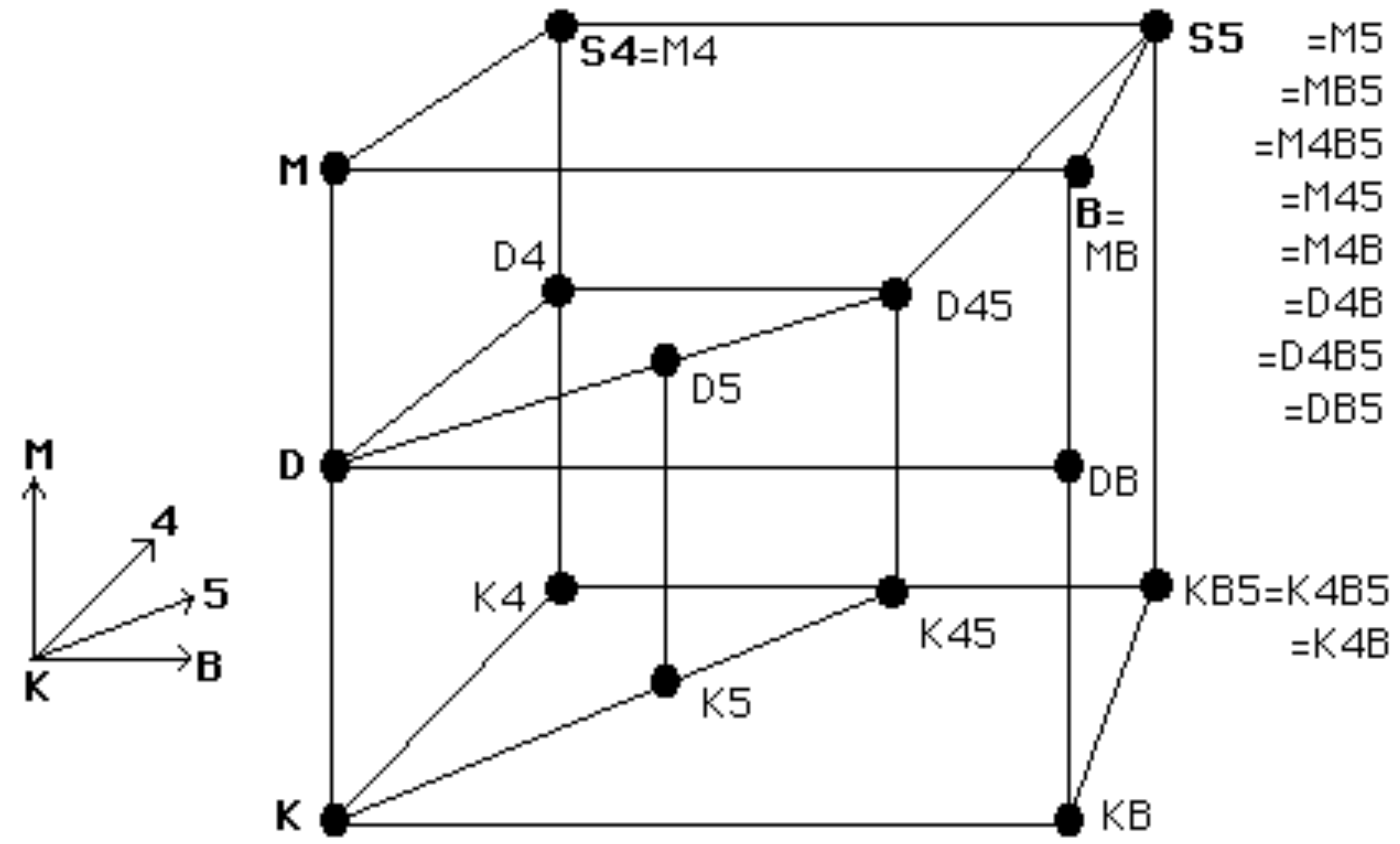


https://plato.stanford.edu/entries/logic-modal/

Non-trivial

Characterisation of modal logics:

• Frame Properties	• Axiom schemes
Reflexive	(M) $\Box A \supset A$
Transitive	(4) $\Box A \supset \Box\Box A$
Euclidean	(5) $\Diamond A \supset \Box\Diamond A$
Symmetric	(B) $A \supset \Box\Diamond A$



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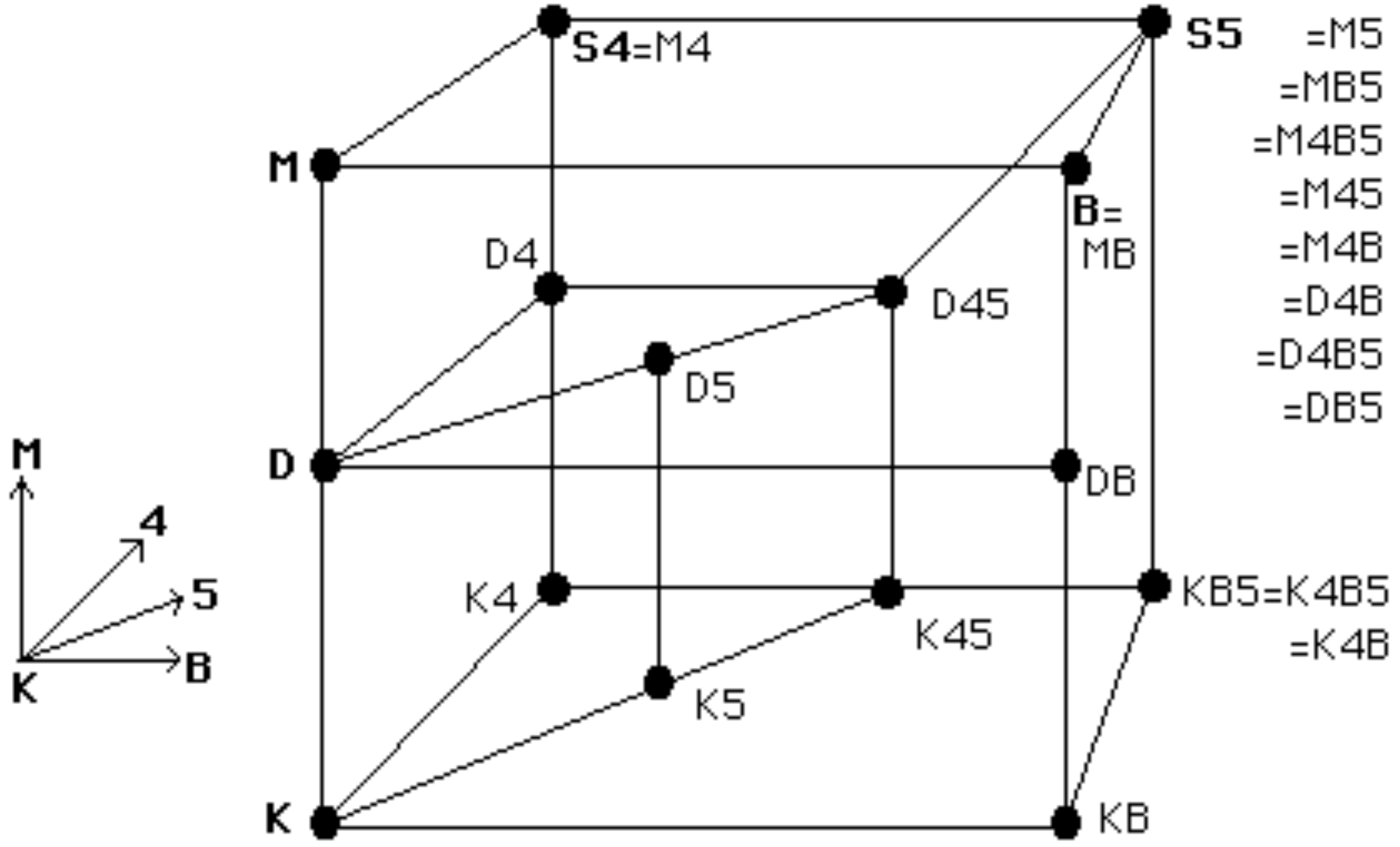
Non-trivial

Characterisation of modal logics:

• Frame Properties	• Axiom schemes
Reflexive Transitive Euclidean Symmetric	$(M)\quad \Box A \supset A$ $(4)\quad \Box A \supset \Box\Box A$ $(5)\quad \Diamond A \supset \Box\Diamond A$ $(B)\quad A \supset \Box\Diamond A$
-	**McKinsey Axiom** $\Box\Diamond A \supset \Diamond\Box A$
Irreflexive $\neg xRx$	-



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Non-trivial

Non-trivial
Multi-Modal Logics with
Interactions

(1) Frame Properties
(2) Axiom Schemes

Interactions

… are axiom schemes that establish the formal relationship between different modal operators in multimodal logics

e.g: $\Box_1 A \supset \Box_2 A$

Non-trivial

Multi-Modal Logics with

Interactions

(1) Frame Properties
(2) Axiom Schemes
(3) Interactions

(1) Frame Properties
(2) Axiom Schemes
(3) Interactions

```
thf(logic_spec, logic, $modal == [
  $designation == $rigid,
  $domains == $constant,
     $modalities == [
        $modal_system_K,
        {$box(#1)} == $modal_system_S4,
        {$box(#2)} ==
           [Frame Properties, Axiom Schemes],
        Interactions, Axiom Schemes ]]).
```

FOML-SPEC

(1) Frame Properties

(2) Axiom Schemes

(3) Interactions

$\neg xRx$

$\Box\Diamond A \supset \Diamond\Box A$

$\Box_1 A \supset \Box_2 A$

FOML-SPEC

(1) Frame Properties

(2) Axiom Schemes

(3) Interactions

$\neg xRx$

```
![X: $ki_world] :
   ~$ki_accessible(X,X))]
```

$\Box \Diamond A \supset \Diamond \Box A$

$\Box_1 A \supset \Box_2 A$

FOML-SPEC

(1) Frame Properties
(2) Axiom Schemes
(3) Interactions

$\neg xRx$

```
![X: $ki_world] :
   ~$ki_accessible(X,X))]
```

$\Box\Diamond A \supset \Diamond\Box A$

```
{$box} @ ({$dia} @ (A))
   => {$dia} @ ({$box} @ (A))
```

$\Box_1 A \supset \Box_2 A$

```
{$box(#1)} @ (A)
   => {$box(#2)} @ (A)
```

FOML-SPEC

HOL

$\neg xRx$

```
![X: $ki_world] :
   ~$ki_accessible(X,X))]
```

$\Box \Diamond A \supset \Diamond \Box A$

```
{$box} @ ({$dia} @ (A))
   => {$dia} @ ({$box} @ (A))
```

$\Box_1 A \supset \Box_2 A$

```
{$box(#1)} @ (A)
   => {$box(#2)} @ (A)
```

$wR_i v \rightarrow \quad r^i_{\mu\rightarrow\mu\rightarrow o} W_\mu V_\mu$

Embedding

FOML-SPEC | HOL

$wR_iv \rightarrow \quad r^i_{\mu\rightarrow\mu\rightarrow o} W_\mu V_\mu$

```
$ki_accessible(W,V)
```

→

```
mrel: (mindex >
(mworld > (mworld
> $o)))
```

```
(mrel @ W) @ V
```

Embedding

FOML-SPEC | HOL

$$\Box_i A \quad \rightarrow \quad \left(\lambda X_{\mu\to o} . \lambda W_{\mu} . \forall V_{\mu} . \neg(r^i\ W\ V) \lor (X\ V)\right)\ \lceil A \rceil$$

Embedding

FOML-SPEC | **HOL**

$$\Box_i A \quad \rightarrow \quad \left(\lambda X_{\mu\to o} . \lambda W_\mu . \forall V_\mu . \neg(r^i\ W\ V) \lor (X\ V)\right)\ \lceil A \rceil$$

```
{box} @ (A)
```

→

```
  mbox: (mindex > ((mworld > $o) > (mworld > $o)))

(mbox = (^ [R:mindex,Phi:(mworld > $o),W:mworld]:
 ((! [V:mworld]: (((((mrel @ R) @ W) @ V)
  => (Phi @ V))))))))
```

```
(mbox @ A)
```

Embedding

FOML-SPEC | HOL

FOML-SPEC	HOL
$\neg xRx$	
`![X: $ki_world] : ~$ki_accessible(X,X))]`	
$\Box\Diamond A \supset \Diamond\Box A$	
`{$box} @ ({$dia} @ (A)) => {$dia} @ ({$box} @ (A))`	
$\Box_1 A \supset \Box_2 A$	
`{$box(#1)} @ (A) => {$box(#2)} @ (A)`	

FOML-SPEC	HOL

$\neg xRx$
`![X: $ki_world] :`
`  ~$ki_accessible(X,X))]`
	`(! [X:mworld]:`
	`  (~ ((((mrel @'#1') @ X) @ X))))))`

$\Box\Diamond A \supset \Diamond\Box A$
`{$box} @ ({$dia} @ (A))`
`  => {$dia} @ ({$box} @ (A))`	

$\Box_1 A \supset \Box_2 A$
`{$box(#1)} @ (A)`
`  => {$box(#2)} @ (A)`	

FOML-SPEC	HOL

$\neg xRx$
`![X: $ki_world] :`
`~$ki_accessible(X,X))]`
	`(! [X:mworld]:`
	`(~ ((((mrel @'#1') @ X) @ X))))))`

$\Box\Diamond A \supset \Diamond\Box A$
`{$box} @ ({$dia} @ (A))`
`=> {$dia} @ ({$box} @ (A))`
	`(! [A:(mworld > $o)]:`
	`((mglobal @(^[W:mworld]:`
	`(((((mbox @'#2')@((mdia@'#2')@A))@ W)`
	`=> (((mdia @‘#2')@((mbox@'#2')@A))@W)))))))`

$\Box_1 A \supset \Box_2 A$
`{$box(#1)} @ (A)`
`=> {$box(#2)} @ (A)`
	`(! [A:(mworld > $o)]:`
	`((mglobal @ (^ [W:mworld]:`
	`(((((mbox @ '#1') @ A) @ W)`
	`=> (((mbox @ '#2') @ A) @ W)))))))`

In Summary:

- The TPTP-Syntax has been extended to allow for the representation of FOML setups characterised by arbitrary **frame properties**, **axiom schemes** and **interactions**
- The implementation of an embedding of such setups into HOL (**Logic Embedding Tool LET**) can be used with ATP systems (**LEO-III**) to reason within these non-trivial logics
- This has (up to our knowledge) not been possible in any existing ATP systems before

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Questions?