VERIFIED VAMPIRE PROOFS IN $\lambda\Pi$ -CALCULUS MODULO

(A MACHINE CHECKABLE PROOF OUTPUT MODE)

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EUROPROOFNET SYMPOSIUM: WORKSHOP ON AUTOMATED REASONING AND PROOF LOGGING, SEPTEMBER 2025

TWO APPROACHES TO FORMAL PROOFS

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Automated theorem prover

(ATP)

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Automated theorem prover

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Interactive theorem prover

(ITP)

AUTOMATED THEOREM PROVER (ATP)

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INPUT

- Axioms
- (negated) conjecture

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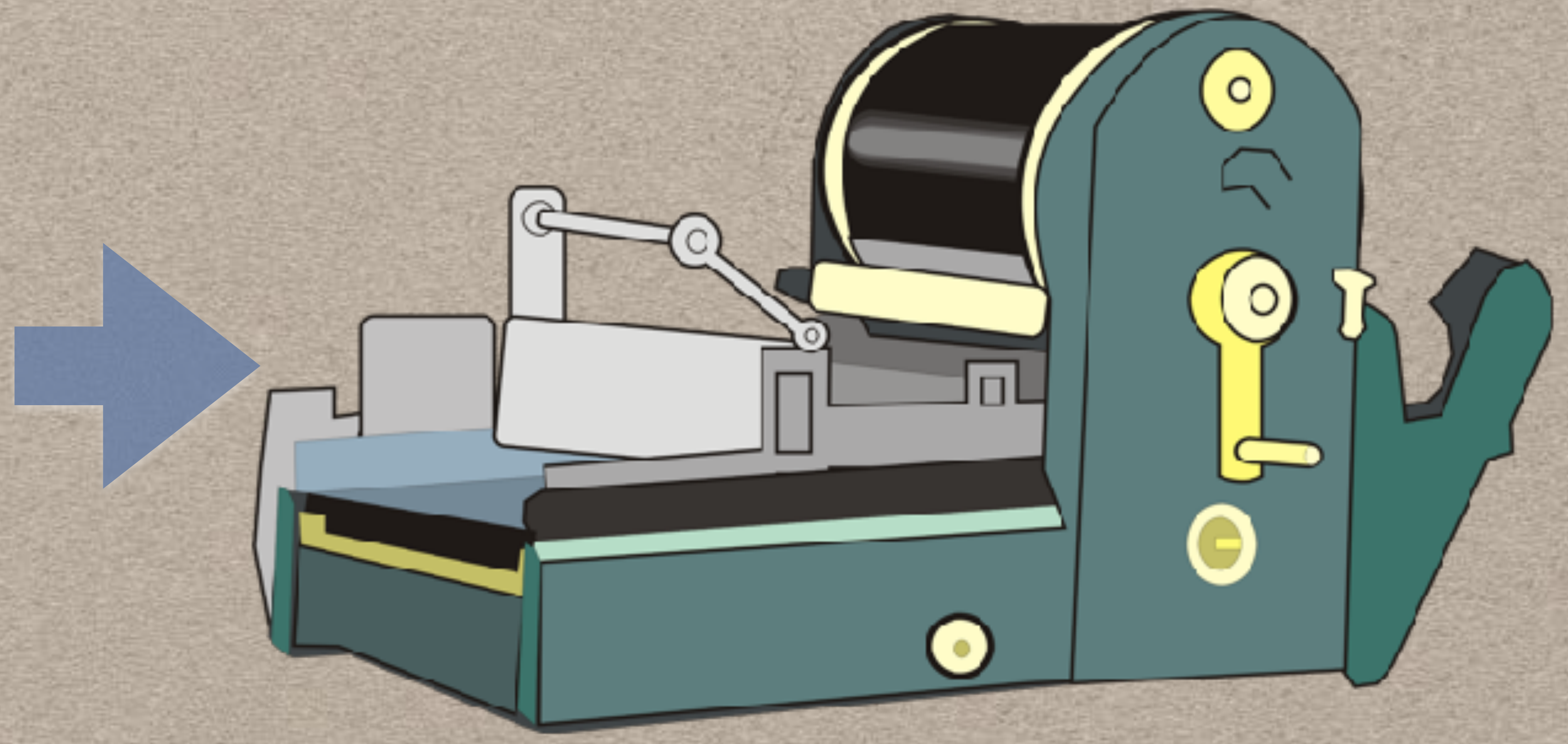

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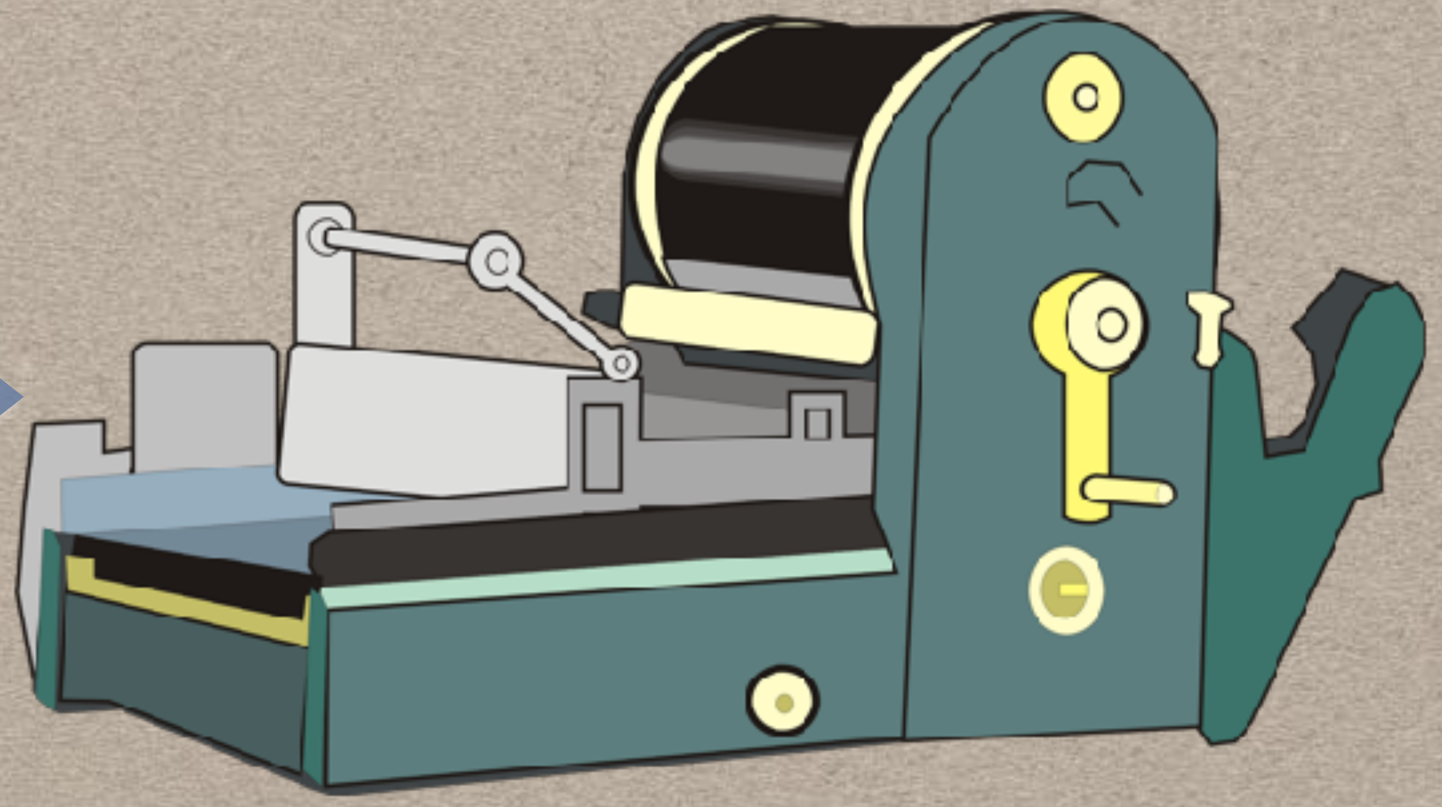

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Proof trace

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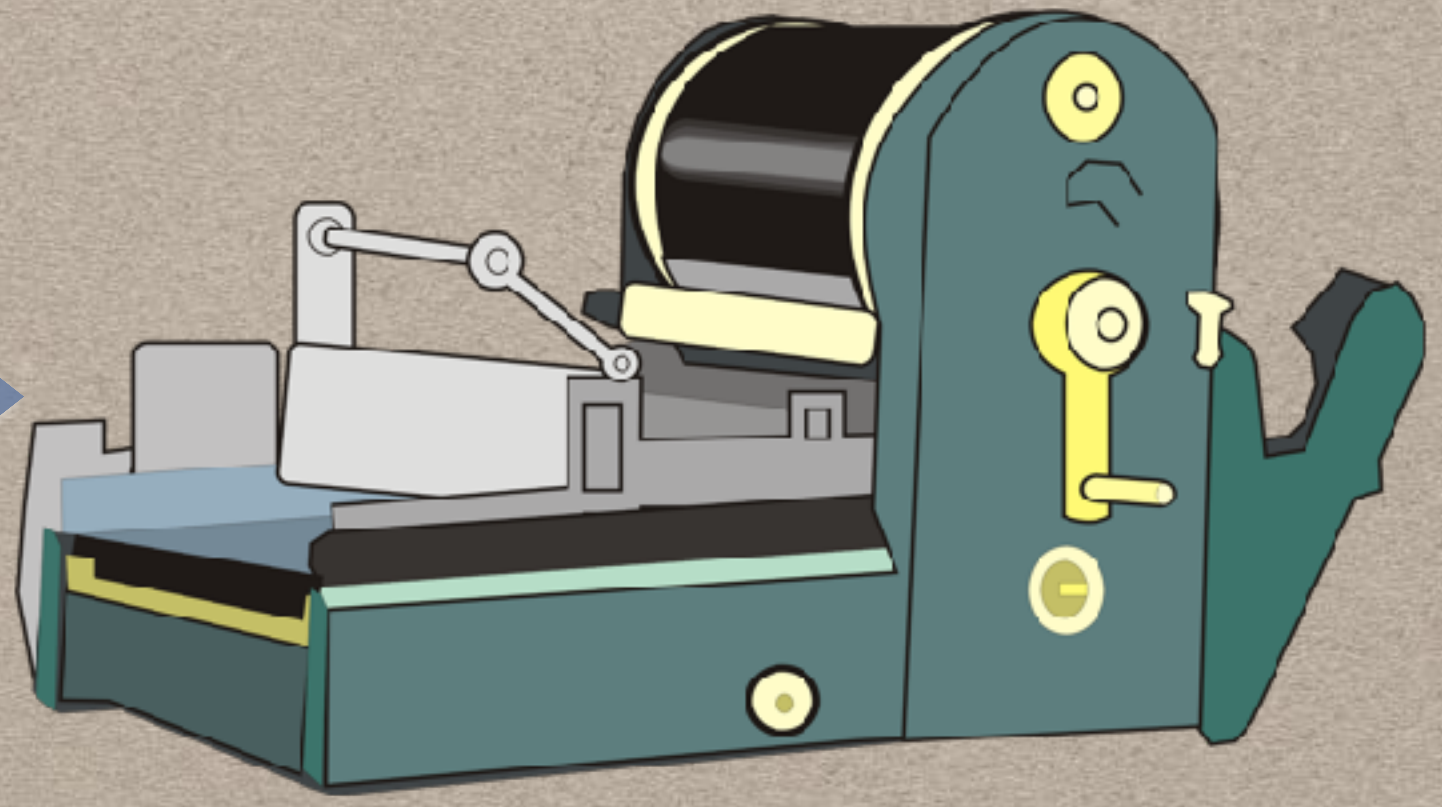

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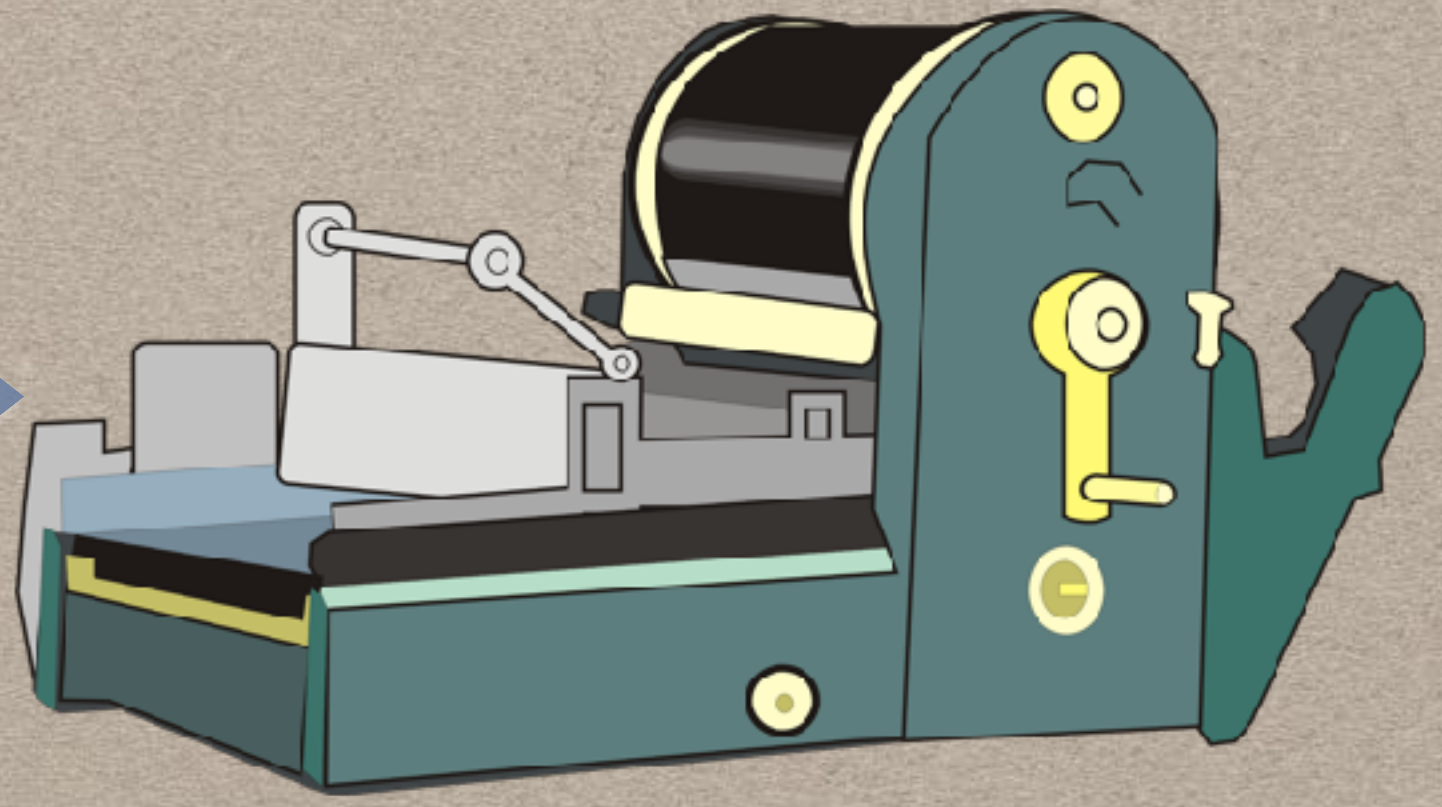

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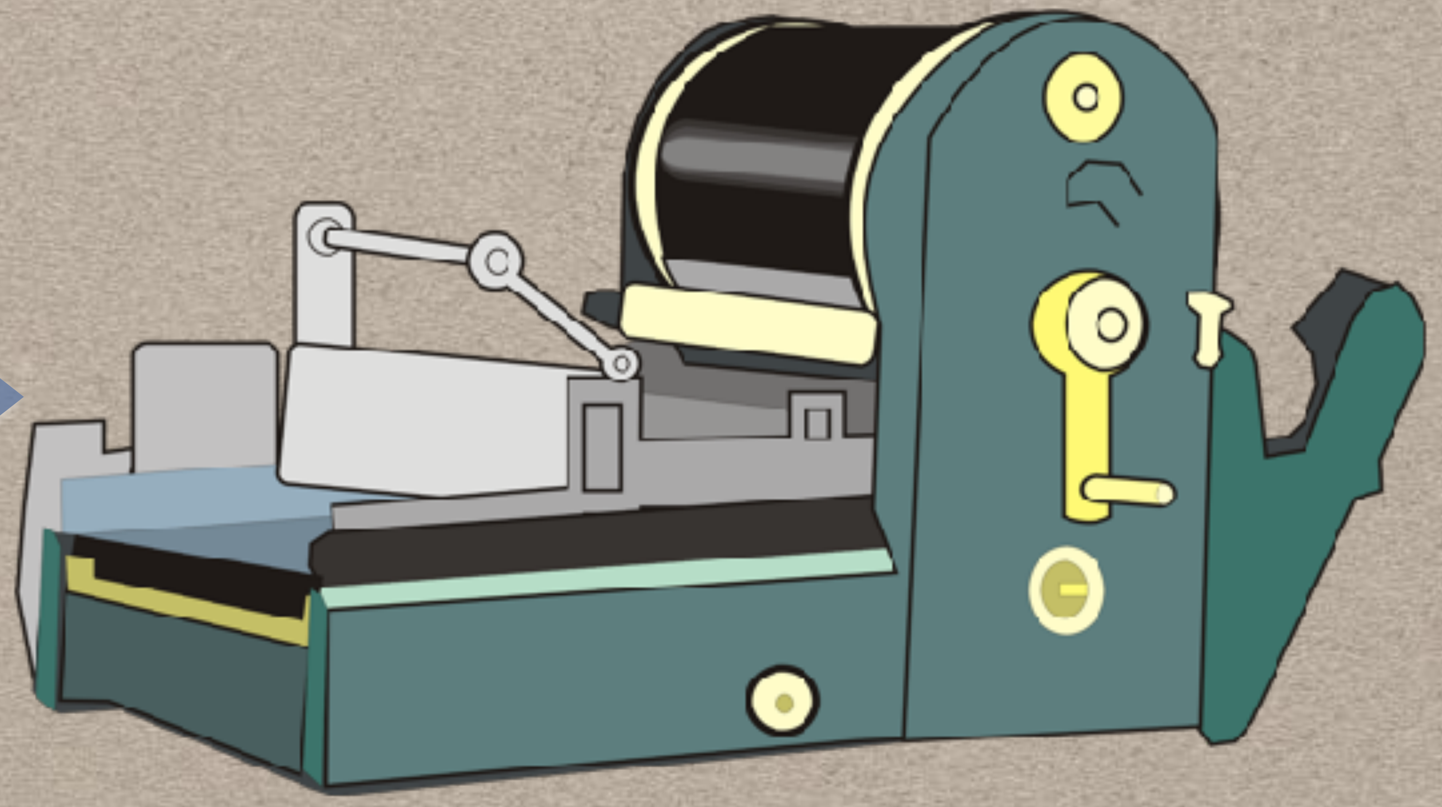

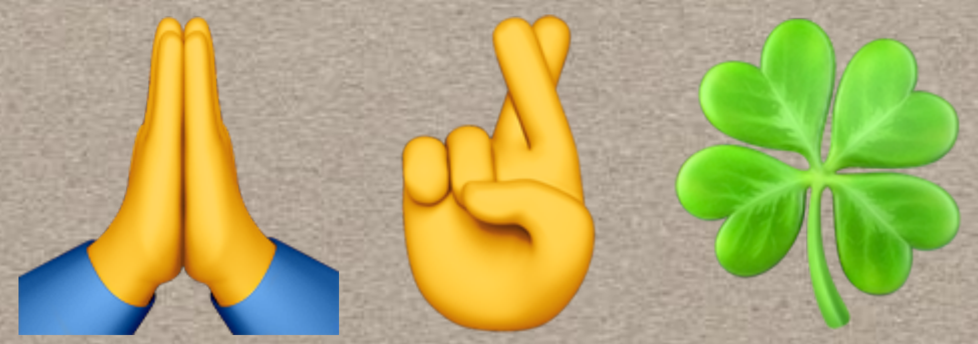

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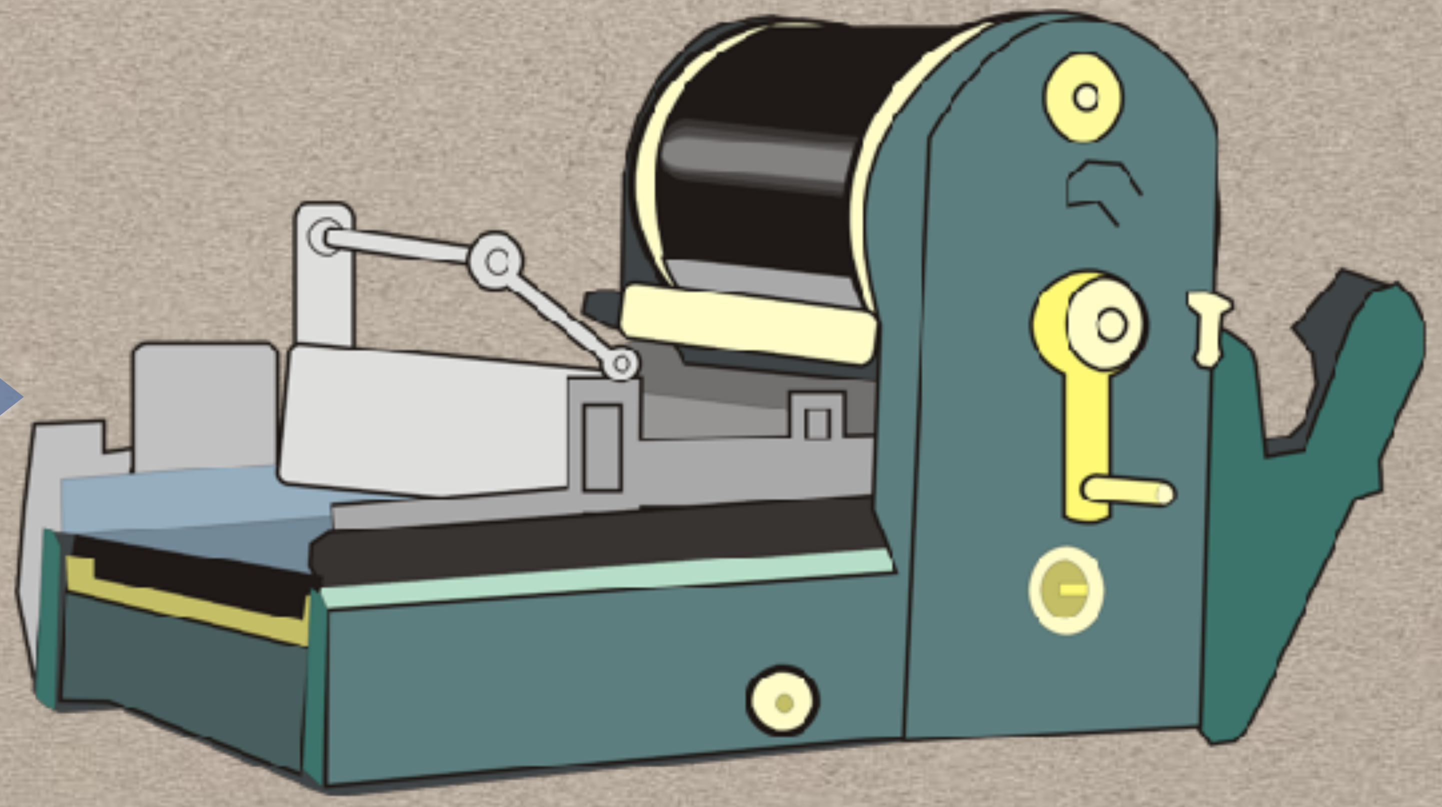

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- Can be used as a "black box"

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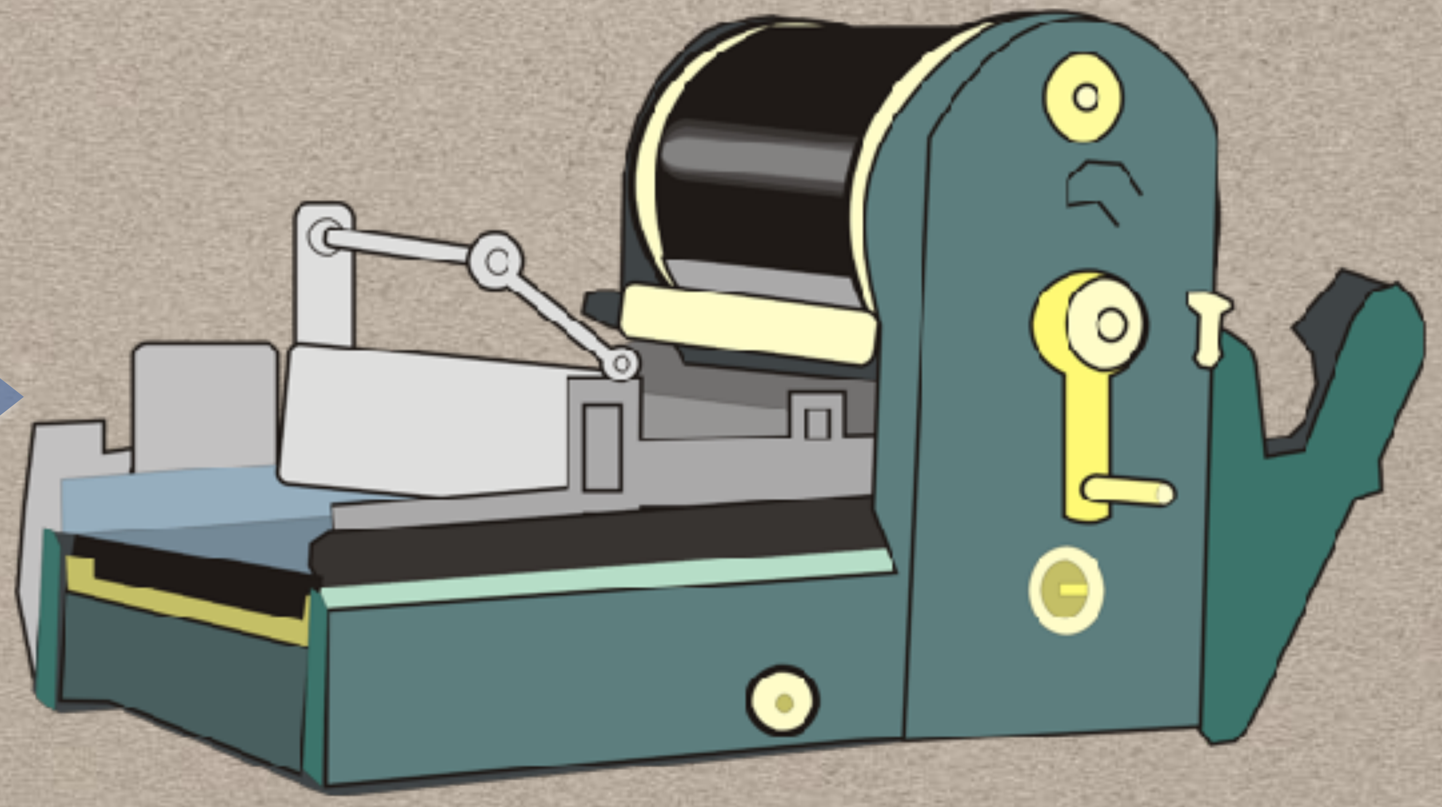

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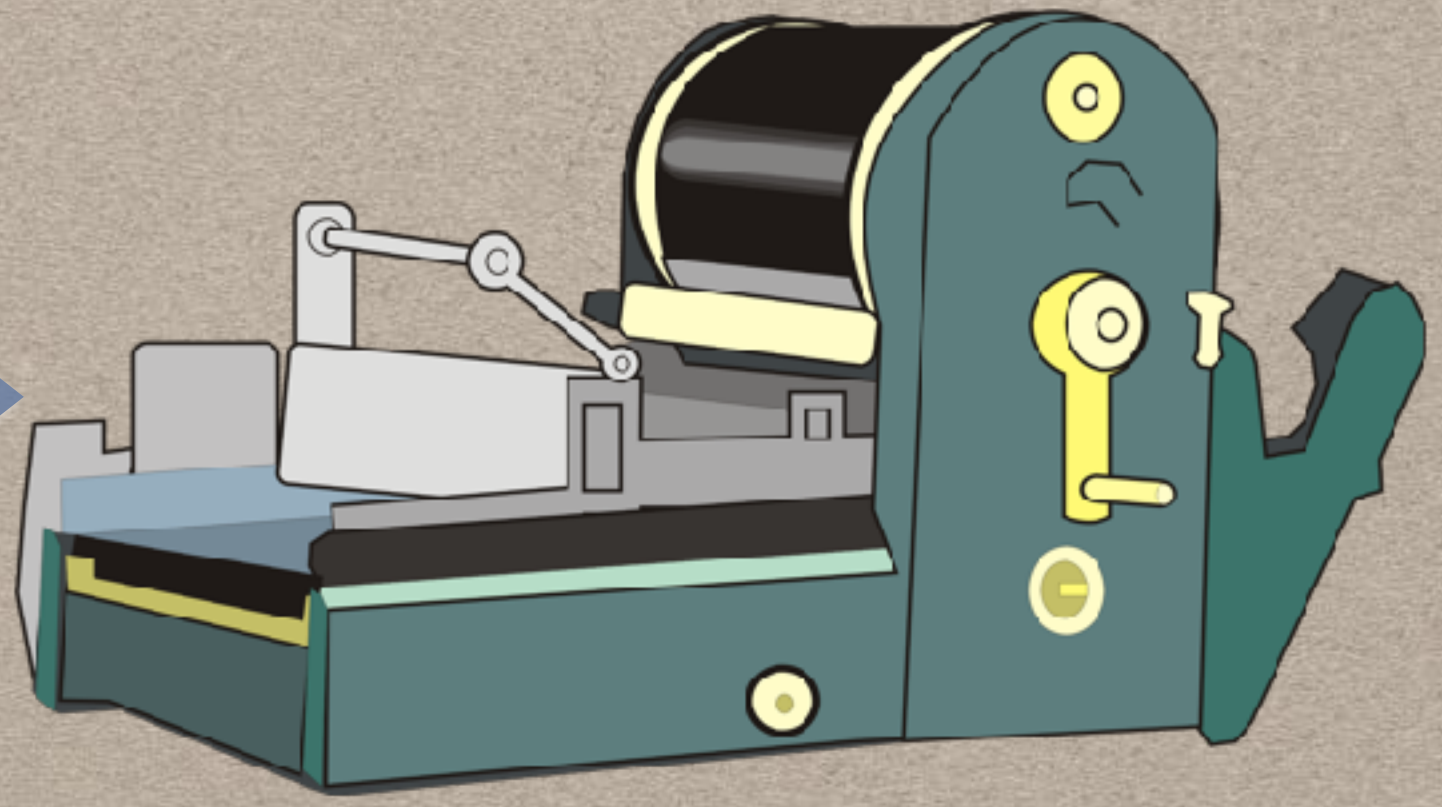

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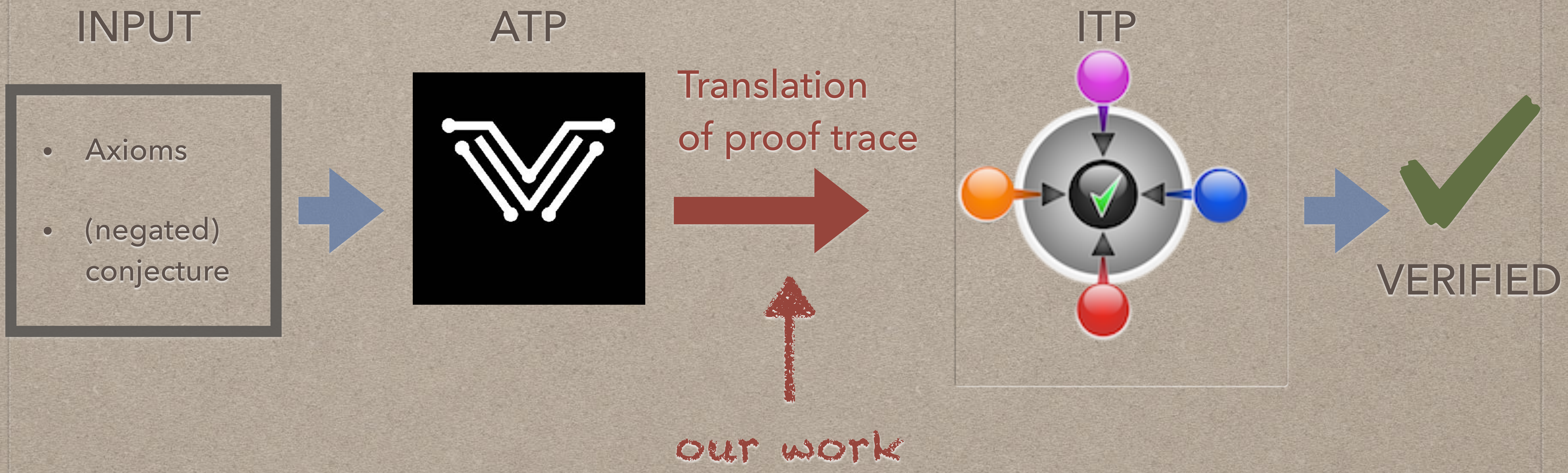
Proof trace

- Can be used as a "black box"
- Complex piece of software
- Big trusted code base

- Sequence / directed graph of (first-order) formulas
- Which premises and inference rules were used
- Should be enough to reconstruct the proof

FROM VAMPIRE TO DEDUKTI



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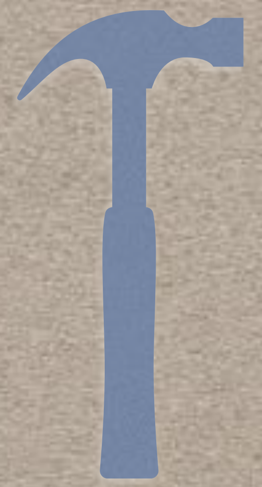
- Interoperability of proofs
- Potential for hammers



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VAMPIRE

- **First-order system**, extended with:
  - ★ reasoning with theories
  - ★ induction
  - ★ higher-order logic
- **Saturation based** theorem prover
- Employs **a number of techniques**: indexing, scheduling, ordered rewriting, AVATAR, heuristics, etc.

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- Scales reasonably well: designed to machine check large proofs
(unlike lambdapi?)

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- Proof format "-p dedukti" (on vampire branch dedukti)

vampire $problem -p dedukti - - proof_extra full | dk check

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vampire $problem -p dedukti - - proof_extra full | dk check

- Using **standard Dedukti encoding of FOL** and Dedukti semantics encode **concrete instances** of Vampire inferences

- Sometimes need to store extra information in the proof: "–proof_extra full"

More on that later

STANDARD ENCODING OF FOL IN DEDUKTI

```
(; Prop ;)
Prop : Type.
def Prf : (Prop -> Type).
true : Prop.
[] Prf true --> (r : Prop -> ((Prf r) -> (Prf r))).
false : Prop.
[] Prf false --> (r : Prop -> (Prf r)).
not : (Prop -> Prop).
[p] Prf (not p) --> ((Prf p) -> (r : Prop -> (Prf r))).
and : (Prop -> (Prop -> Prop)).
[p, q] Prf (and p q) --> (r : Prop -> (((Prf p) -> ((Prf q) -> (Prf r))) -> (Prf r))).
or : (Prop -> (Prop -> Prop)).
[p, q] Prf (or p q) --> (((Prf p) -> (Prf false)) -> (((Prf q) -> (Prf false)) -> (Prf false))).
imp : (Prop -> (Prop -> Prop)).
[p, q] Prf (imp p q) --> ((Prf p) -> (Prf q)).
iff : Prop -> Prop -> Prop.
[p, q] Prf (iff p q) --> (Prf (and (imp p q) (imp q p))).

(; Set ;)
Set : Type.
injective El : (Set -> Type).
iota : Set.
inhabit : A : Set -> El A.

(; Equality ;)
def eq : a : Set -> El a -> El a -> Prop.
[a, x, y] Prf (eq a x y) --> p : (El a -> Prop) -> Prf (p x) -> Prf (p y).
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```
(; Quant ;)
forall : (a : Set -> (((El a) -> Prop) -> Prop)).
[a, p] Prf (forall a p) --> (x : (El a) -> (Prf (p x))).
exists : (a : Set -> (((El a) -> Prop) -> Prop)).
[a, p] Prf (exists a p) --> (r : Prop -> ((x : (El a) -> ((Prf (p x)) -> (Prf r))) -> (Prf r))).

(; polymorphic quantifier ;)
forall_poly : (Set -> Prop) -> Prop.
[p] Prf (forall_poly p) --> a : Set -> Prf (p a).
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Polymorphic Leibniz encoding of equality.

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(; Clauses ;)
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def ec : prop_clause.
def cons : (Prop -> (prop_clause -> prop_clause)).
def clause : Type.
def cl : (prop_clause -> clause).
def bind : (A : Set -> (((El A) -> clause) -> clause)).
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def Prf_prop_clause : (prop_clause -> Type).

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"empty" AVATAR clause is regular clause.

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All this rewrites/normalizes to encoding of FOL.

We did not introduce any new axioms.

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  - Superposition
  - Subsumption resolution
  - Equality resolution
  - AVATAR
  - Trivial equality removal
  - Factoring
  - Remove duplicate literals
  - Some pre-processing steps
    - ★ Equality resolution with deletion
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  - AVATAR ← Presented later
  - Trivial equality removal
  - Factoring
  - Remove duplicate literals
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    - ★ Definiton unfolding
- **Many sorted logic and polymorphism** are supported

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Full first-order formulas (FOF) can be parsed, but clausification steps are not checked.

- Limited pre-processing: clausification is not checked
- No higher-order reasoning and no theories yet
- Turn off all the fun bits (only inferences on the previous slide)
  - ★ But inferences are done **incrementally** :)
  - ★ Unsupported inferences are handled by "**sorry**".

We emit a warning during type-checking when there is a sorry.

SOME LESSONS LEARNED DURING IMPLEMENTATION

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- **Equality is symmetric**:
  - Vampire will switch LHS and RHS when convenient.
  - Manually insert commutativity lemmas during proof-printing.
- **Numbering deduction steps accordingly**: we give the deduction steps in Deduct script the same number as appears in the “default” Vampire proof trace, to ease uncovering bugs (if the elaborated proofs do not type check).

AVATAR

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- **Splitting** clauses and **offloading** the disjunctive structure **to a SAT solver**
- For proof logging: **introducing propositional labels**

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Definition: propositional label to a sub-clause

$$\mathsf{sp}_1 \equiv \forall xy.\ P(x, f(y)) \vee \neg Q(y)$$

$$\mathsf{sp}_2 \equiv \forall z.\ c = z$$

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Split: clauses split into variable-disjoint components, deriving SAT clause

$$\frac{\neg Q(z) \vee c = y \vee P(x, f(z))}{\mathsf{sp}_1 \vee \mathsf{sp}_2}$$

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Definition: propositional label to a sub-clause

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Refutation: last step of the proof, derives false, because SAT set is unsatisfiable.

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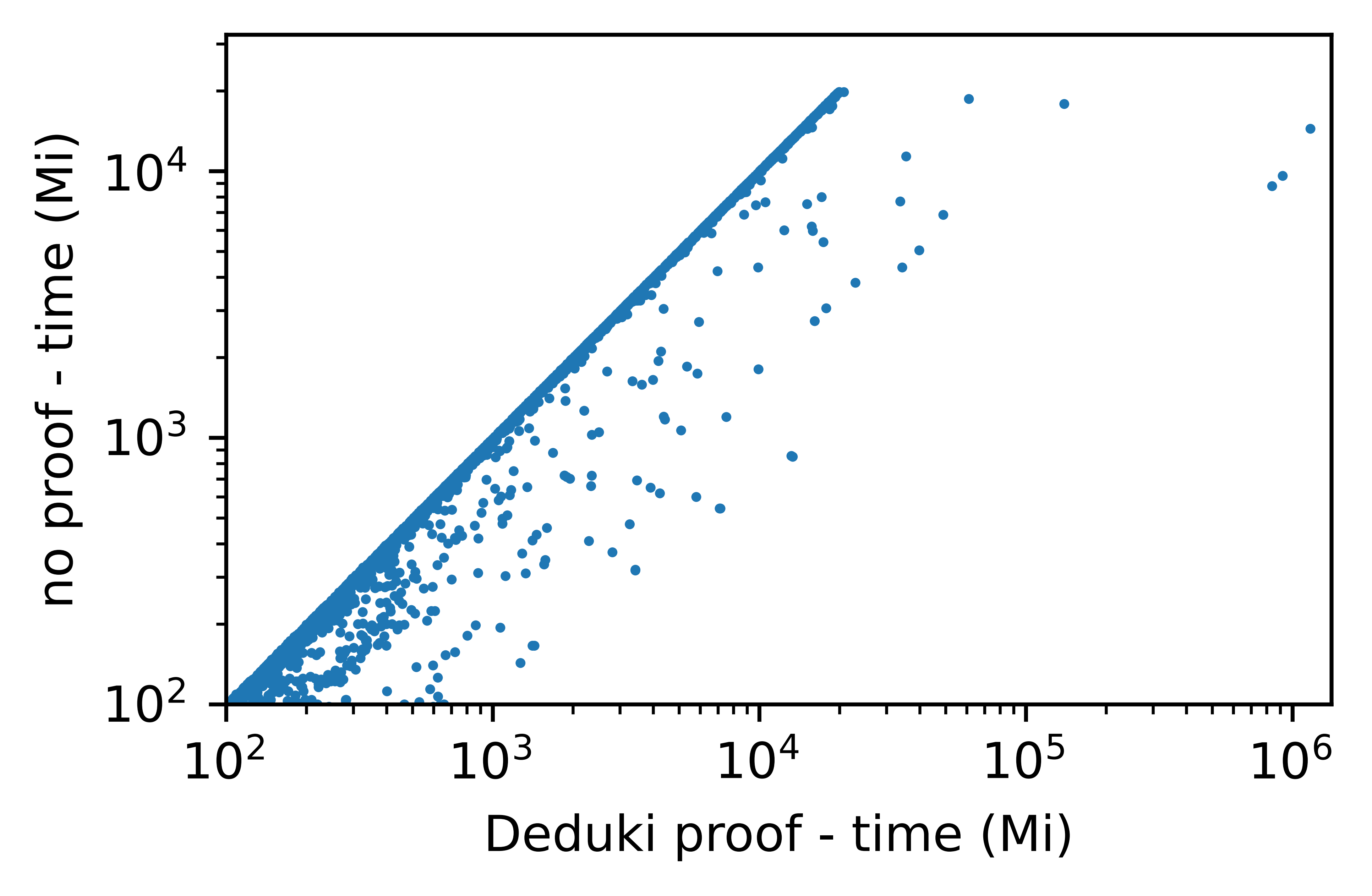
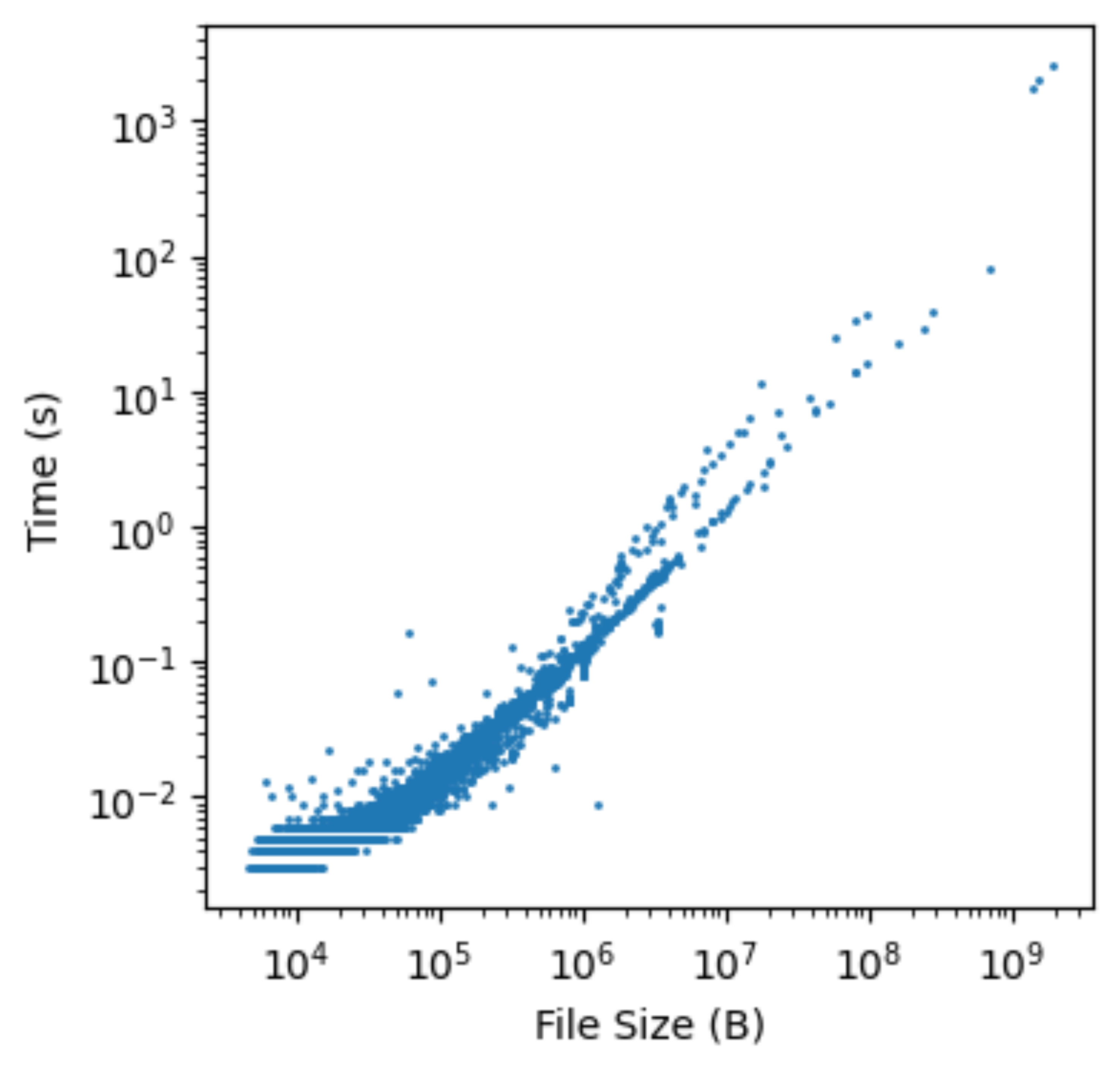
- **All proofs were successfully checked by Dedukti** (increasing stack was necessary)


no proof - time (Mi)
Deduki proof - time (Mi)


Time (s)
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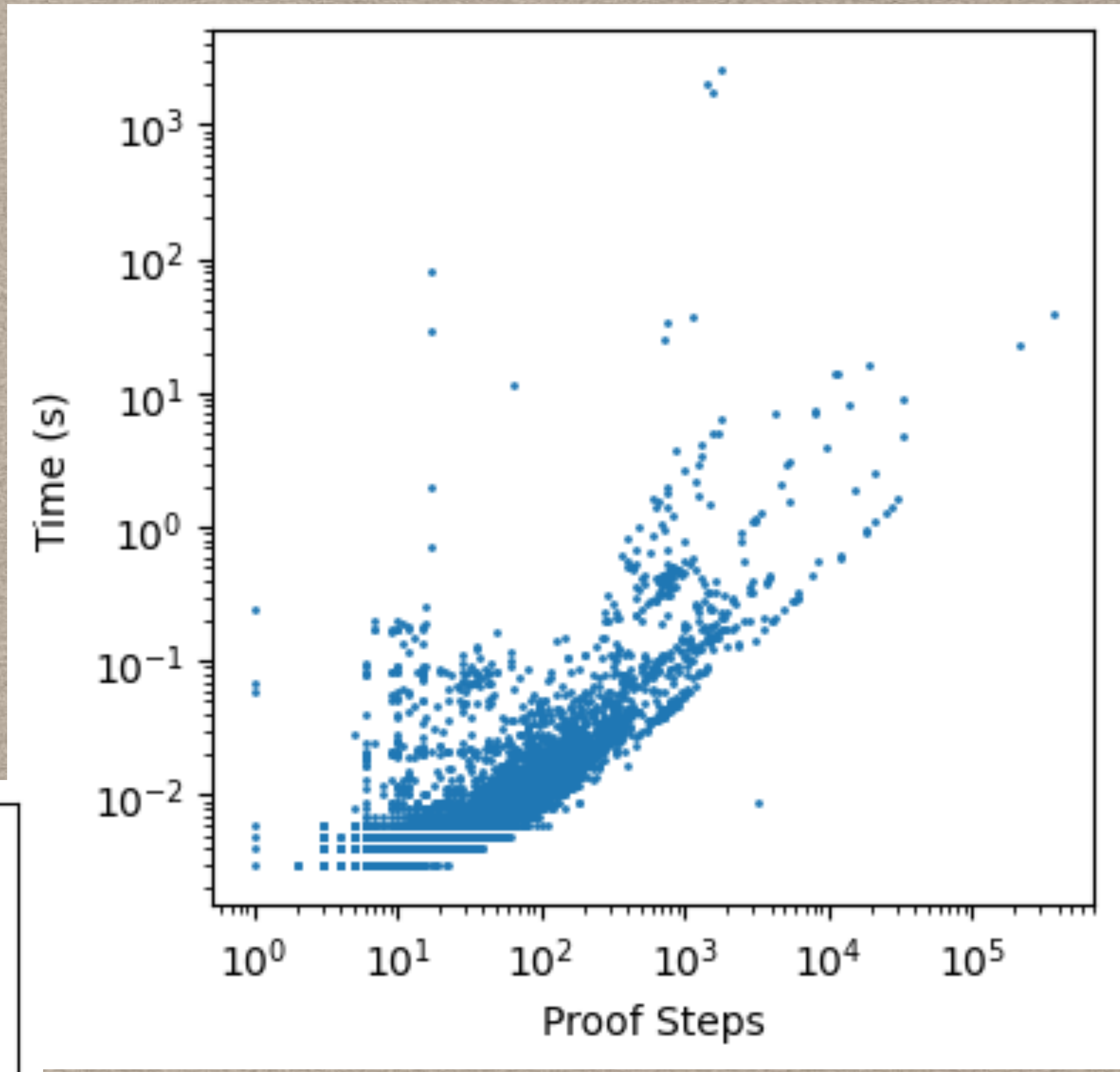
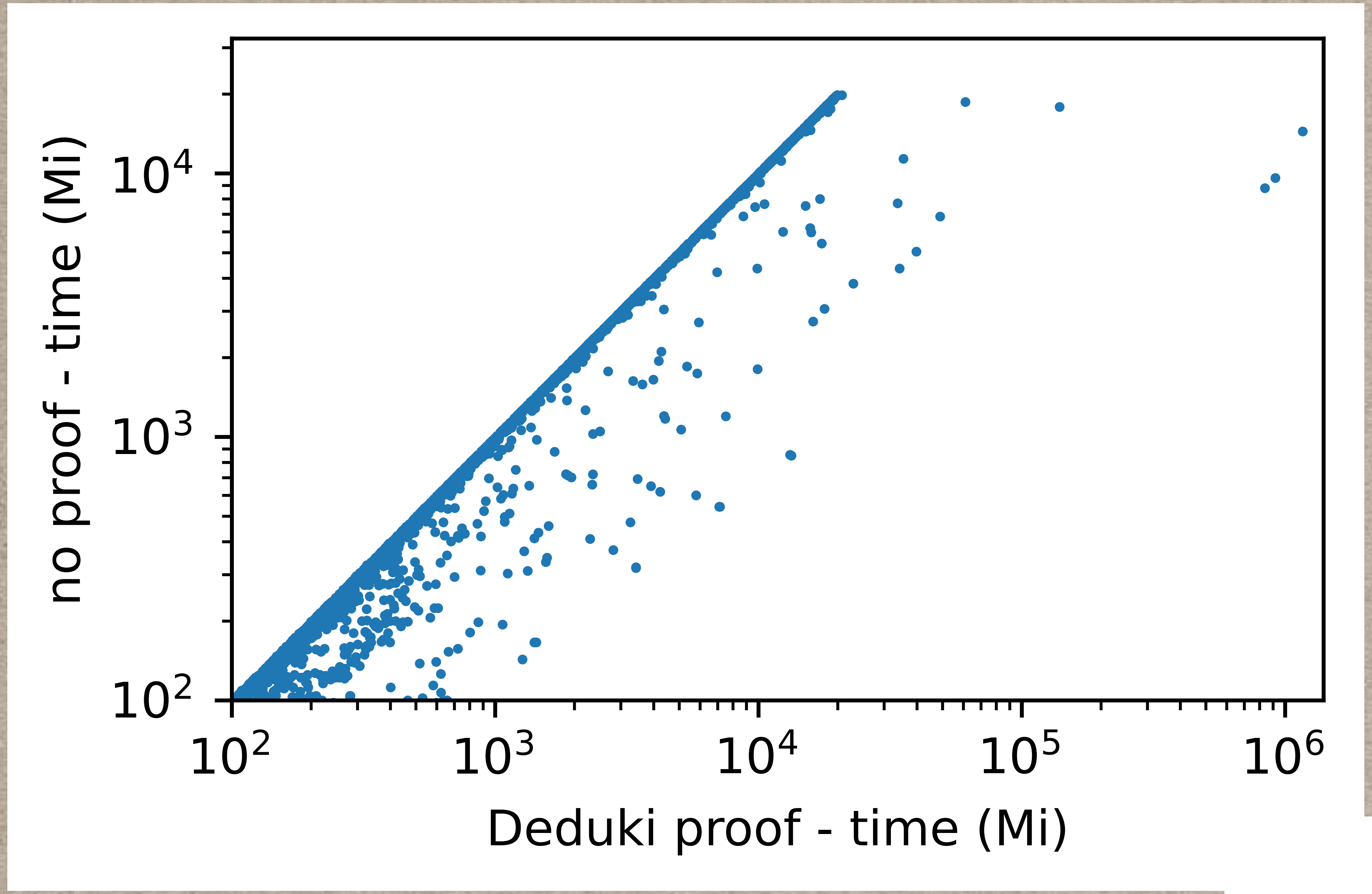
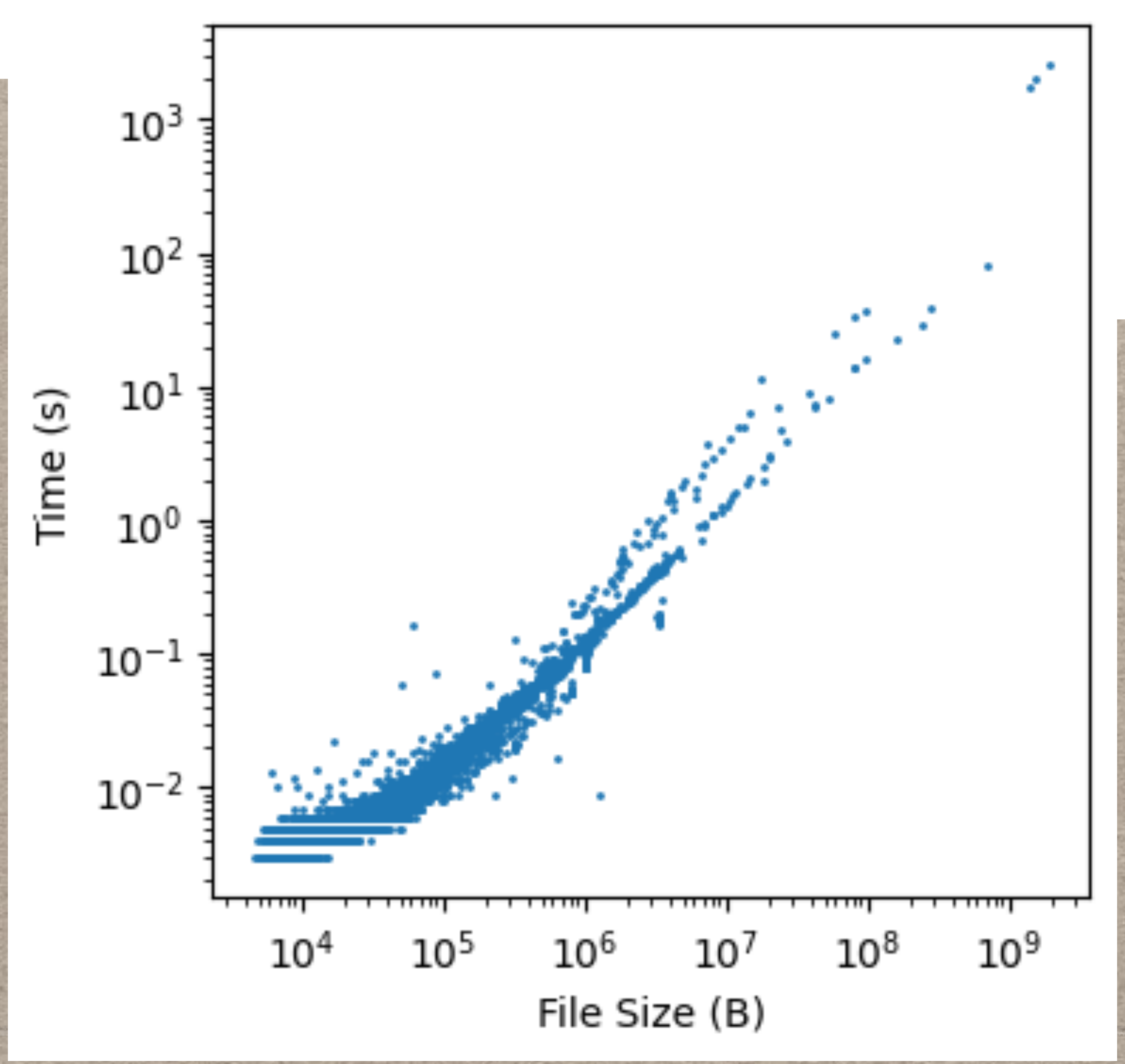
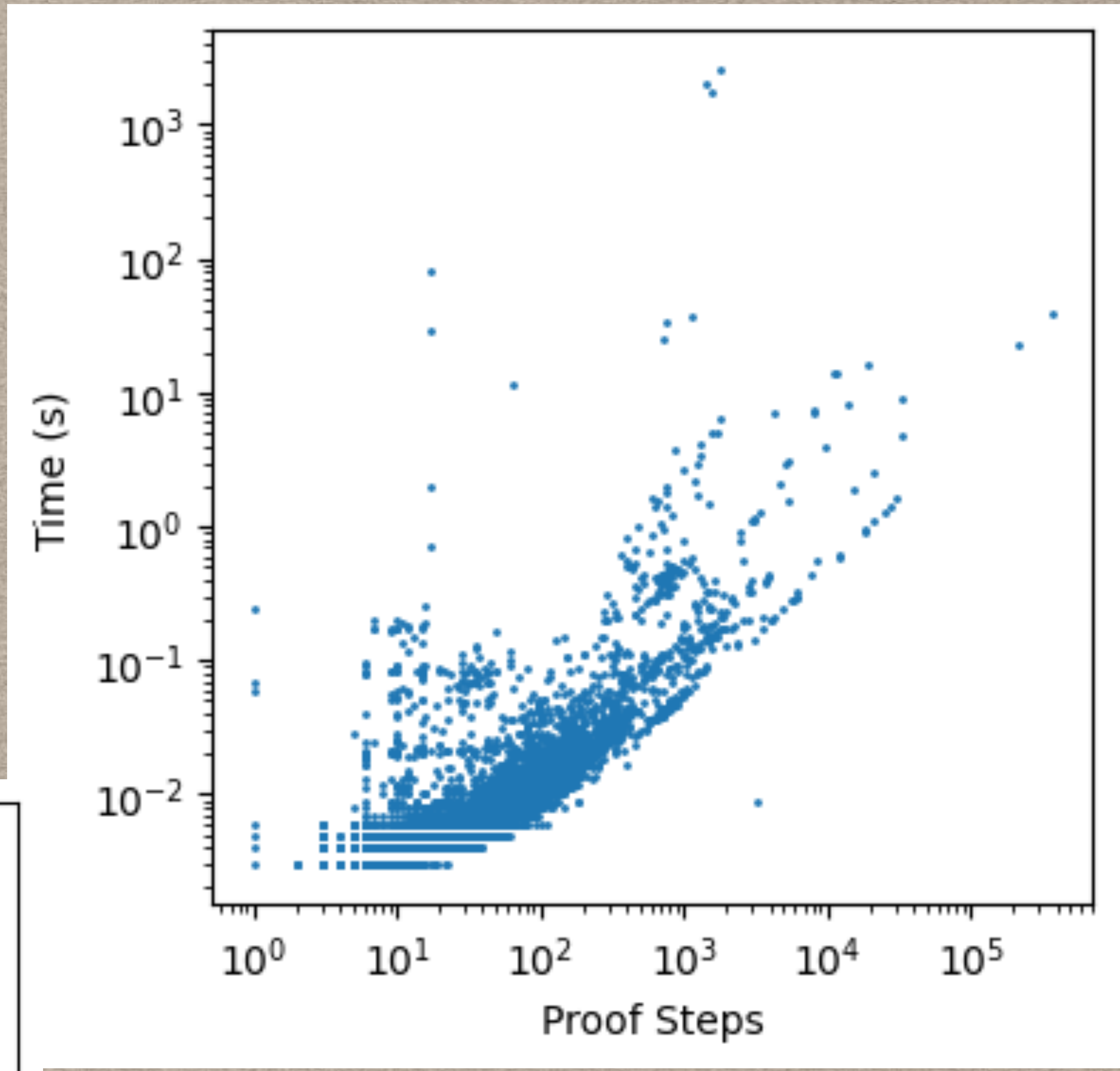

Time (s)
Proof Steps







Proof checking (dk check) time

Median time: 0.006s

Average time: 0.881s

Max time: 2567s (42min)

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- Vampire outputs the proof of double negation:
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- The last step to go from double negation to asserting conjecture is classical, but we actually do not explicitly do that.

Note: Due to double-negation translation, it is always possible to have a classical proof, that is constructive all but for the one (last) step. This is an illustration of this fact.

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- **Polarity flip**: exploiting Dedukti definitions
  def polarity_flip (p : Prop -> Prop) := (not p) and proceeding with polarity_flip(p).

QUESTIONS?

vampire $problem -p dedukti - - proof_extra full

| egrep -v ^%

| dk check /dev/stdin