Techniques and Tools for Automated Termination Analysis

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COST Action EuroProofNet, WG3 meeting, Feb 2023

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WG3 deliverables

- Month 18: Comparison of the approaches used in the international Software Verification competition SV-COMP.
- Month 24: Software prototype for the automated inference of program specifications as logical axioms.
- Month 48: Collection of verification challenges with summary of working recipes for verifying them.
- Month 48: Technique for syntax-semantics interface for program verification with or without type systems.

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⇒ Some thoughts on **competitions** and approaches for **termination**

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- Expected output
  (claim with no proof, human-readable proof, machine-checkable proof)

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$\longrightarrow$ Safety properties, at SV-COMP since 2012, for C and Java programs

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$\Rightarrow$ tools use automatable **sufficient criteria**

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- Different competitions may make different assumptions
  (...which make sense in context)

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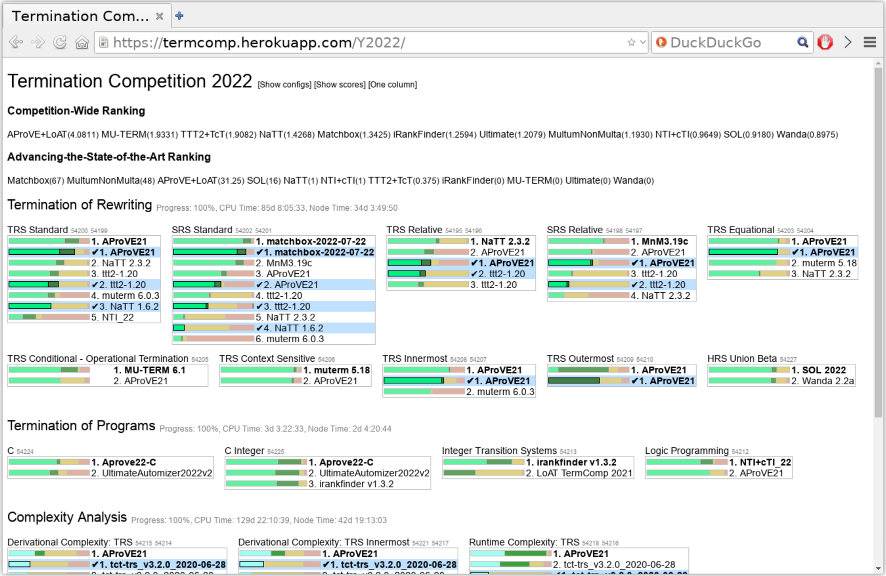
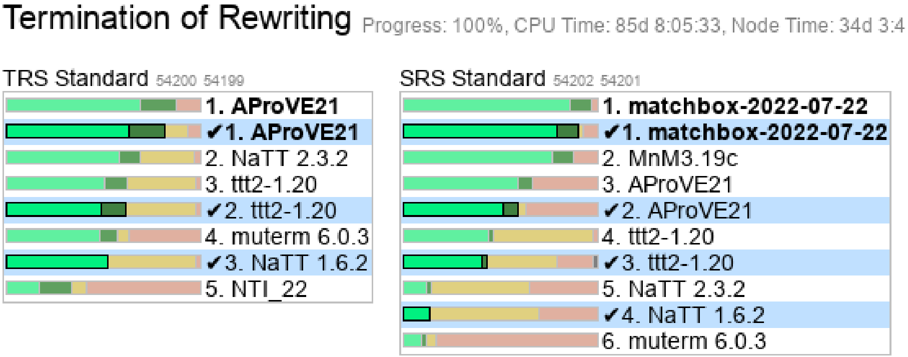
- SV-COMP: other participating tools, for **frontend languages**

termCOMP with certification (✓) (1/2)



Let's zoom in . . .


Termination of Rewriting
Progress: 100%, CPU Time: 85d 8:05:33, Node Time: 34d 3:4
TRS Standard 54200 54199
1. AProVE21
✔1. AProVE21
2. NaTT 2.3.2
3. ttt2-1.20
✔2. ttt2-1.20
4. muterm 6.0.3
✔3. NaTT 1.6.2
5. NTI_22
SRS Standard 54202 54201
1. matchbox-2022-07-22
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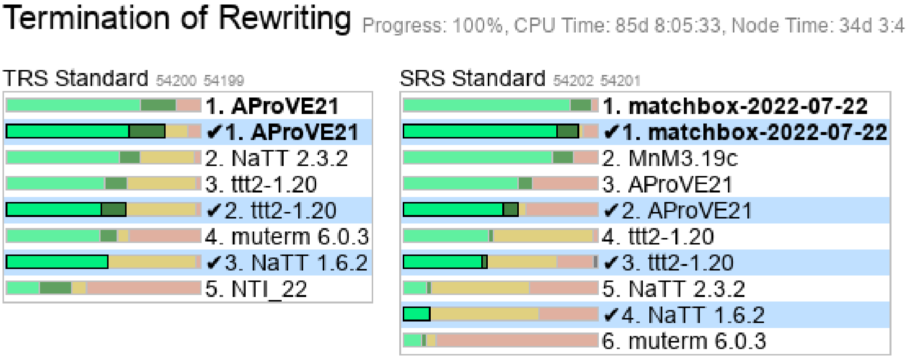

⇒ proof certification is competitive!

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Example (Termination can be simple)

while $x > 0$:
$x = x - 1$

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- **Repeat** until the whole program is proved terminating

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if $(\mathsf{x} \geq 0)$
  while $(\mathsf{x} \neq 0)$
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Does this program terminate?
(x ranges over $\mathbb{Z}$)

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$$\begin{array}{lcll} \ell_0(x) & \rightarrow & \ell_1(x) & [x \geq 0] \\ \ell_0(x) & \rightarrow & \ell_3(x) & [x < 0] \\ \ell_1(x) & \rightarrow & \ell_2(x) & [x \neq 0] \\ \ell_2(x) & \rightarrow & \ell_1(x-1) & \\ \ell_1(x) & \rightarrow & \ell_3(x) & [x = 0] \end{array}$$

And this one?

Oh no! $\ell_1(-1) \rightarrow \ell_2(-1) \rightarrow \ell_1(-2) \rightarrow \ell_2(-2) \rightarrow \ell_1(-3) \rightarrow \cdots$

Papers on termination of imperative programs often about **integers** as data

Example (Imperative Program)

ℓ_0: **if** $(x \geq 0)$
ℓ_1: **while** $(x \neq 0)$
ℓ_2: $x = x - 1;$

Does this program terminate?
(x ranges over $\mathbb{Z}$)

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$\Rightarrow$ **Restrict initial states to** $\ell_0(z)$ for $z \in \mathbb{Z}$

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$\Rightarrow$ **Restrict initial states** to $\ell_0(z)$ for $z \in \mathbb{Z}$

$\Rightarrow$ Find **invariant** $x \geq 0$ at ℓ_1, ℓ_2 (how?)

Proving termination with invariants

Example (Transition system with invariants)

$$
\begin{array}{lcll}
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Prove termination by ranking function $[\,\cdot\,]$ with $[\ell_0](x) = [\ell_1](x) = \cdots = x$

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$$[\ell_0](x) = a_0 + b_0 \cdot x, \quad [\ell_1](x) = a_1 + b_1 \cdot x, \quad \ldots$$

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Constraints here:

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Use Farkas' Lemma to eliminate $\forall x$, solver for **linear** constraints gives model for a_i, b_i.

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Searching for invariants using SMT

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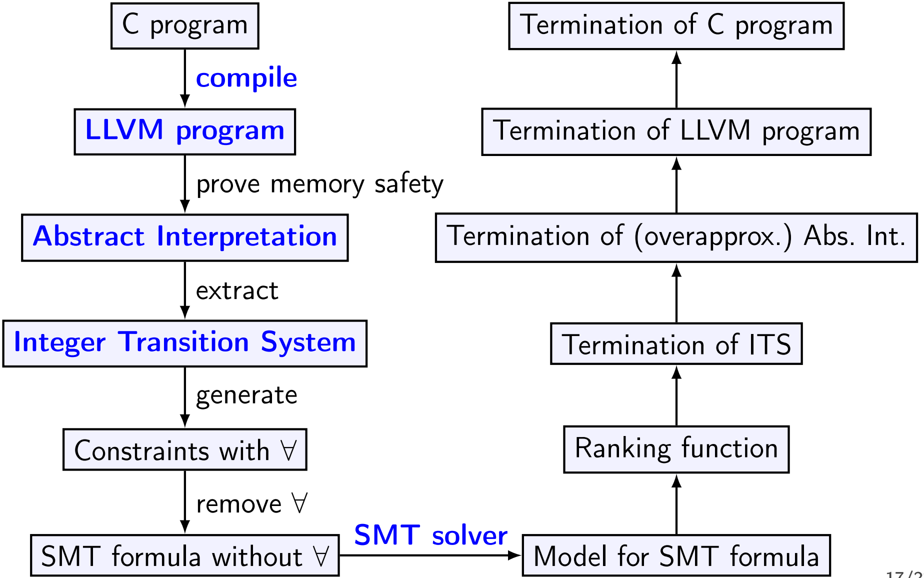
Nowadays all SMT-based!

WG3: what is interesting for our tool inventory?

Example: AProVE's termination analysis for C **in detail**



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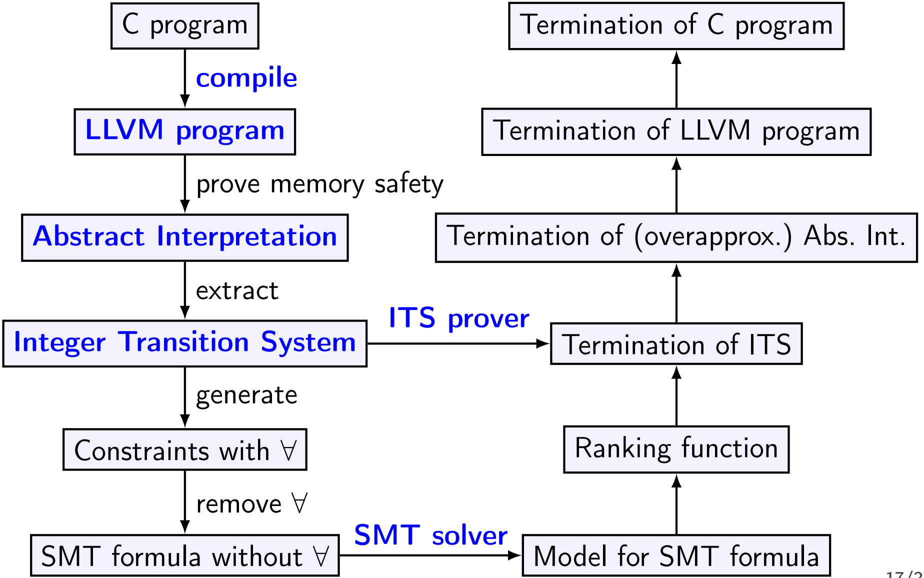

Towards the first deliverable

Tool inventory of verification tools

- What inputs are supported?

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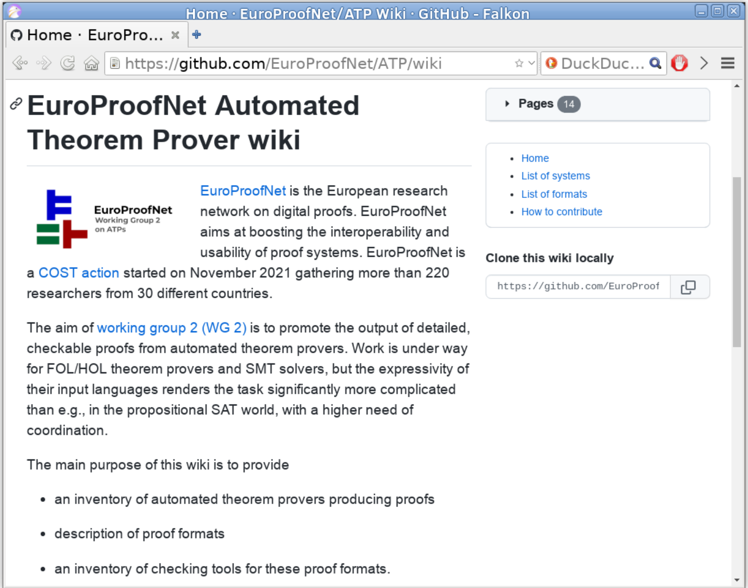
Inspiration: WG2's inventory of ATPs (1/3)

Inspiration: https://github.com/EuroProofNet/ATP/wiki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EuroProofNet Automated Theorem Prover wiki

EuroProofNet
Working Group 2
on ATPs

EuroProofNet is the European research network on digital proofs. EuroProofNet aims at boosting the interoperability and usability of proof systems. EuroProofNet is a COST action started on November 2021 gathering more than 220 researchers from 30 different countries.

The aim of working group 2 (WG 2) is to promote the output of detailed, checkable proofs from automated theorem provers. Work is under way for FOL/HOL theorem provers and SMT solvers, but the expressivity of their input languages renders the task significantly more complicated than e.g., in the propositional SAT world, with a higher need of coordination.

The main purpose of this wiki is to provide

- an inventory of automated theorem provers producing proofs
- description of proof formats
- an inventory of checking tools for these proof formats.

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- List of systems
- List of formats
- How to contribute

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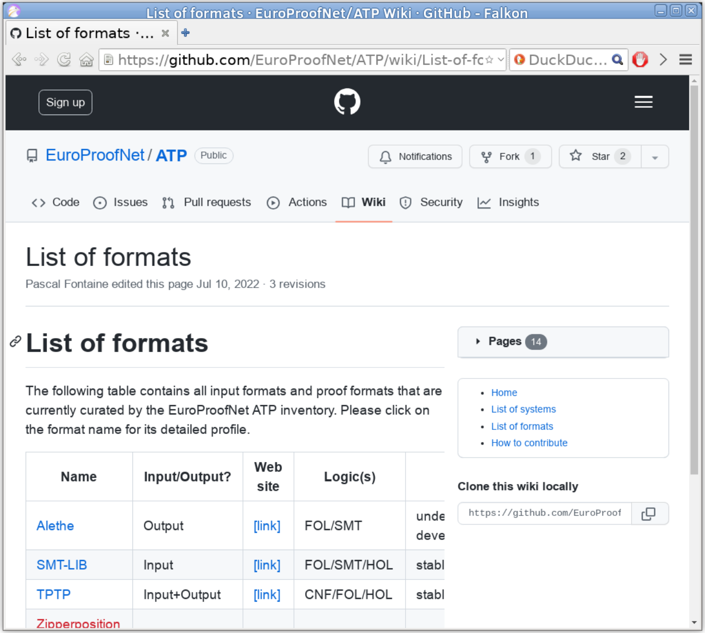
https://github.com/EuroProof

Inspiration: WG2's inventory of ATPs (2/3)



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List of formats

Pascal Fontaine edited this page Jul 10, 2022 · 3 revisions

List of formats

The following table contains all input formats and proof formats that are currently curated by the EuroProofNet ATP inventory. Please click on the format name for its detailed profile.

Name	Input/Output?	Web site	Logic(s)	
Alethe	Output	[link]	FOL/SMT	unde deve
SMT-LIB	Input	[link]	FOL/SMT/HOL	stabl
TPTP	Input+Output	[link]	CNF/FOL/HOL	stabl
Zipperposition				

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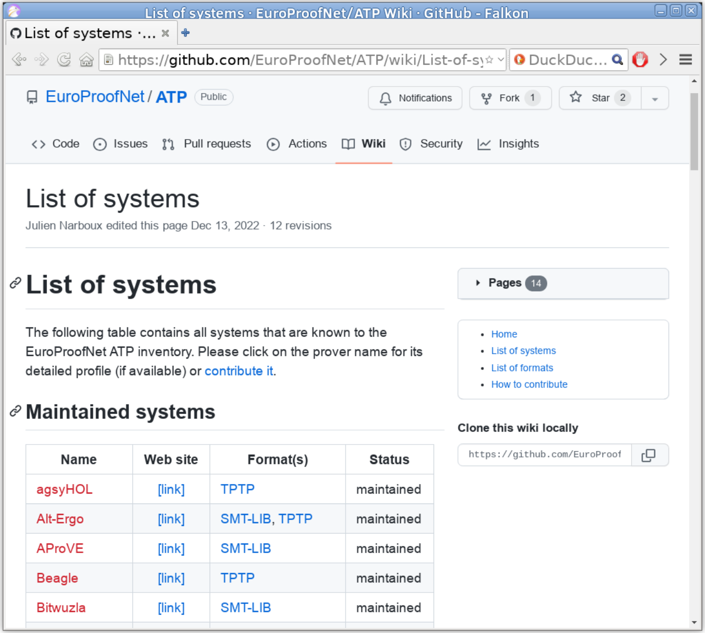
Inspiration: WG2's inventory of ATPs (3/3)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List of systems

Julien Narboux edited this page Dec 13, 2022 · 12 revisions

List of systems

The following table contains all systems that are known to the EuroProofNet ATP inventory. Please click on the prover name for its detailed profile (if available) or contribute it.

Maintained systems

Name	Web site	Format(s)	Status
agsyHOL	[link]	TPTP	maintained
Alt-Ergo	[link]	SMT-LIB, TPTP	maintained
AProVE	[link]	SMT-LIB	maintained
Beagle	[link]	TPTP	maintained
Bitwuzla	[link]	SMT-LIB	maintained

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Conclusion: towards the first deliverable

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