Finding Facts in Large Formalization Libraries: Two Isabelle/AFP Attempts

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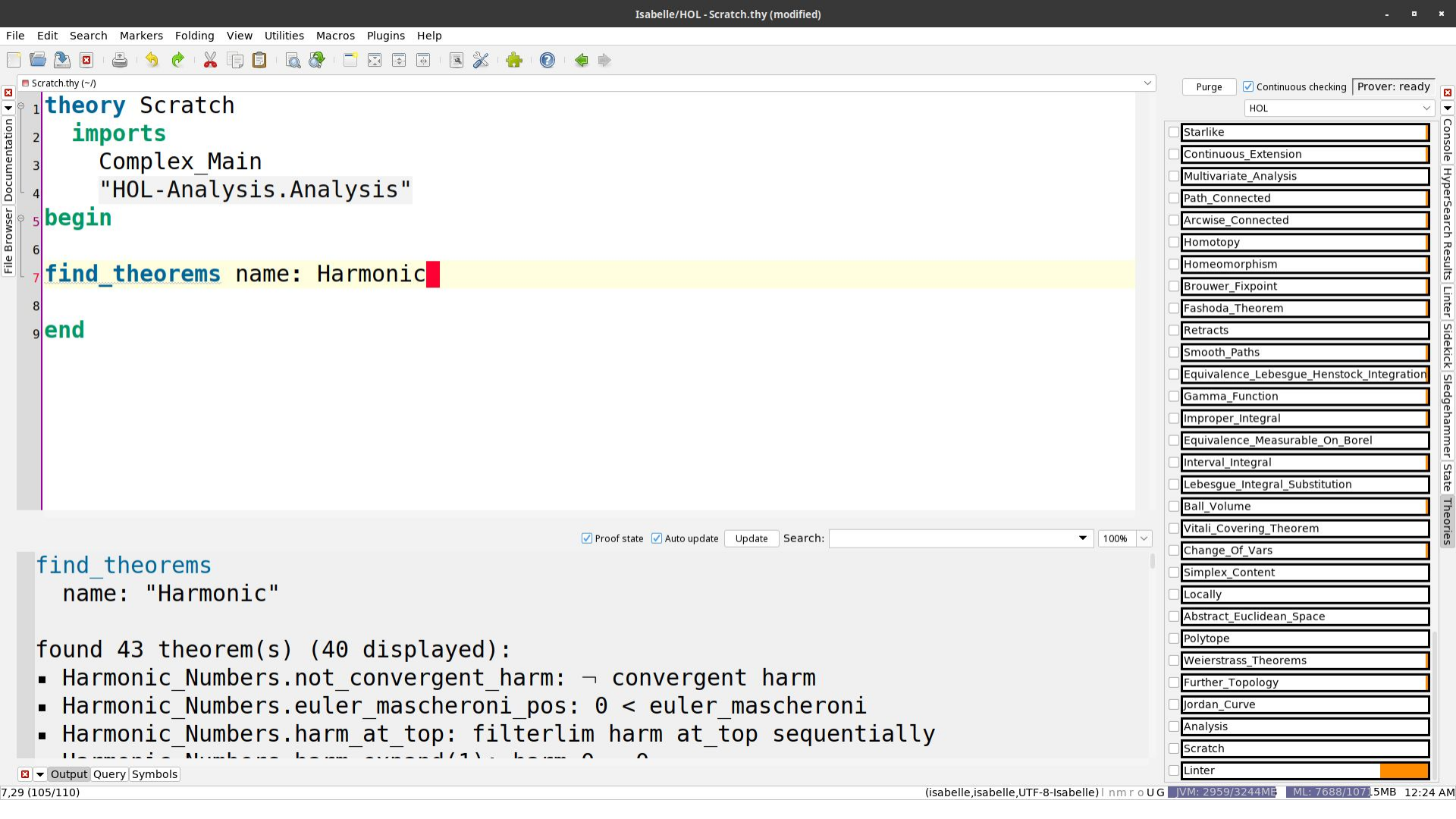
In the beginning… there was find_theorems

Isabelle/HOL - Scratch.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
Scratch.thy (~/)
theory Scratch
imports
Complex_Main
"HOL-Analysis.Analysis"
begin
find_theorems Harmonic
end
Purge
Continuous checking
Prover: ready
HOL
Starlike
Continuous_Extension
Multivariate_Analysis
Path_Connected
Arcwise_Connected
Homotopy
Homeomorphism
Brouwer_Fixpoint
Fashoda_Theorem
Retracts
Smooth_Paths
Equivalence_Lebesgue_Henstock_Integration
Gamma_Function
Improper_Integral
Equivalence_Measurable_On_Borel
Interval_Integral
Lebesgue_Integral_Substitution
Ball_Volume
Vitali_Covering_Theorem
Change_Of_Vars
Simplex_Content
Locally
Abstract_Euclidean_Space
Polytope
Weierstrass_Theorems
Further_Topology
Jordan_Curve
Analysis
Scratch
Linter
Console HyperSearch Results Linter Sidekick Sledgehammer State Theories
File Browser Documentation
Proof state
Auto update
Update
Search:
100%
find_theorems
"Harmonic"
found nothing
Output Query Symbols
7,23 (99/104)
(isabelle,isabelle,UTF-8-Isabelle)
JVM: 2755/3244MB
ML: 7640/10702MB
12:21 AM


Isabelle/HOL - Scratch.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
Scratch.thy (~/)
theory Scratch
imports
Complex_Main
"HOL-Analysis.Analysis"
begin
find_theorems name: Harmonic
end
Purge
Continuous checking
Prover: ready
HOL
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Locally
Abstract_Euclidean_Space
Polytope
Weierstrass_Theorems
Further_Topology
Jordan_Curve
Analysis
Scratch
Linter
Proof state
Auto update
Update
Search:
100%
find_theorems
name: "Harmonic"
found 43 theorem(s) (40 displayed):
Harmonic_Numbers.not_convergent_harm: ¬ convergent harm
Harmonic_Numbers.euler_mascheroni_pos: 0 < euler_mascheroni
Harmonic_Numbers.harm_at_top: filterlim harm at_top sequentially
Output
Query
Symbols
7,29 (105/110)
(isabelle,isabelle,UTF-8-Isabelle)
JVM: 2959/3244MB
ML: 7688/10715MB
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Isabelle/HOL - Scratch.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
Scratch.thy (~/)
theory Scratch
imports
Complex_Main
"HOL-Analysis.Analysis"
begin
find_theorems name: harmonic
end
Purge
Continuous checking
Prover: ready
HOL
Starlike
Continuous_Extension
Multivariate_Analysis
Path_Connected
Arcwise_Connected
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Jordan_Curve
Analysis
Scratch
Linter
Console
HyperSearch Results
Linter
Sidekick
Sledgehammer
State
Theories
Documentation
File Browser
Proof state
Auto update
Update
Search:
100%
find_theorems
name: "harmonic"
found 3 theorem(s):
Summation_Tests.not_summable_harmonic: ¬ summable (λn. inverse (of_nat n))
Harmonic_Numbers.alternating_harmonic_series_sums:
(λk. (- 1) ^ k / real (Suc k)) sums ln 2
Output
Query
Symbols
7,29 (105/110)
(isabelle,isabelle,UTF-8-Isabelle)
JVM: 2690/3244MB
ML: 7640/10702MB
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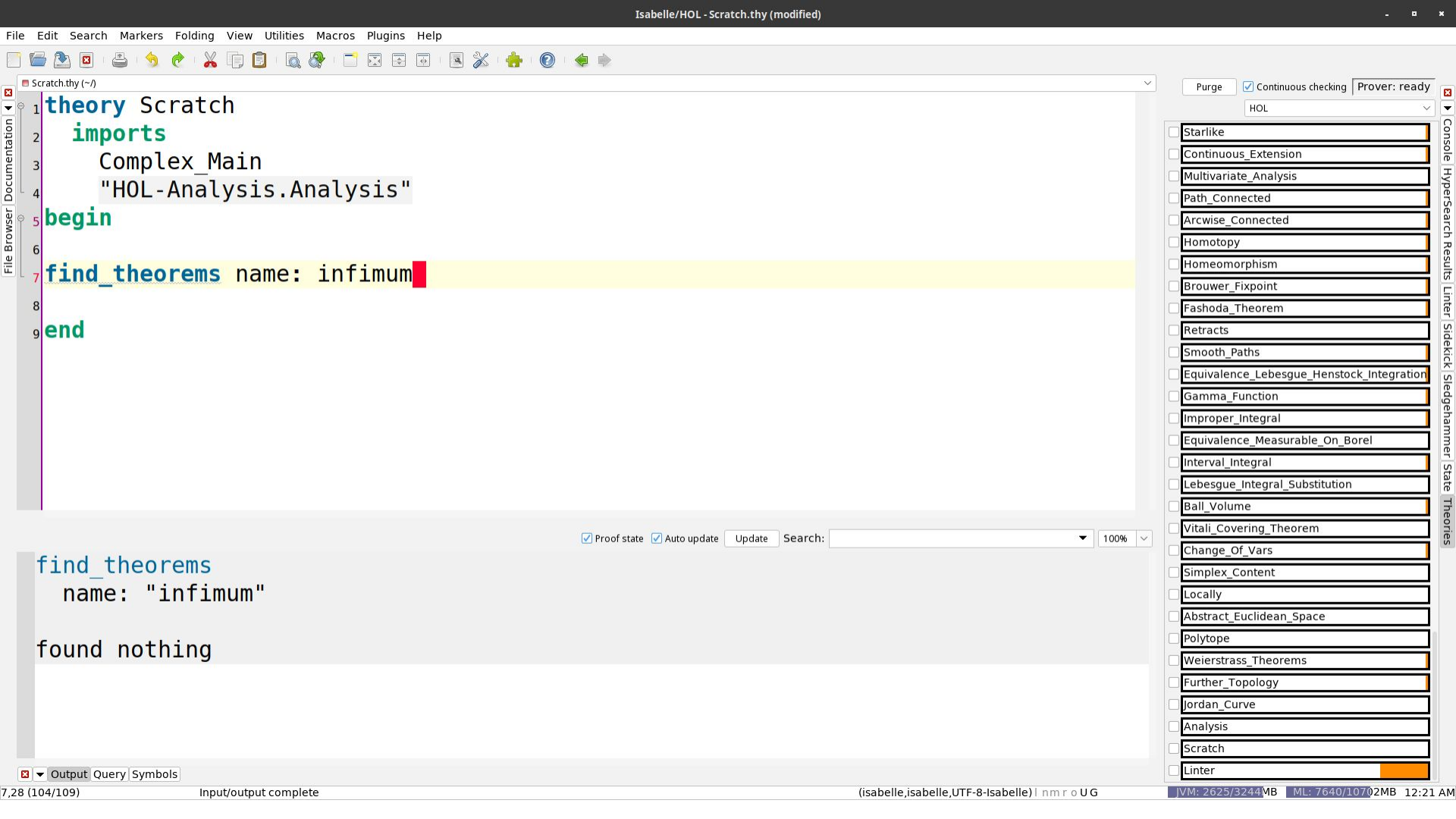
Isabelle/HOL - Scratch.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
Scratch.thy (~/)
theory Scratch
imports
Complex_Main
"HOL-Analysis.Analysis"
begin
find_theorems name: infimum
end
Purge
Continuous checking
Prover: ready
HOL
Starlike
Continuous_Extension
Multivariate_Analysis
Path_Connected
Arcwise_Connected
Homotopy
Homeomorphism
Brouwer_Fixpoint
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Further_Topology
Jordan_Curve
Analysis
Scratch
Linter
Console HyperSearch Results Linter Sidekick Sledgehammer State Theories
File Browser Documentation
Proof state
Auto update
Update
Search:
100%
find_theorems
name: "infimum"
found nothing
Output Query Symbols
7,28 (104/109)
Input/output complete
(isabelle,isabelle,UTF-8-Isabelle)
JVM: 2625/3244MB
ML: 7640/10702MB
12:21 AM

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begin
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Linter
Sidekick
Sledgehammer
State
Theories
File Browser
Documentation
Proof state
Auto update
Update
Search:
100%
find_theorems
name: "Infimum"
found nothing
Output
Query
Symbols
7,28 (104/109)
(isabelle,isabelle,UTF-8-Isabelle)
JVM: 2807/3244MB
ML: 7640/10702MB
12:21 AM

Isabelle/HOL - Scratch.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
Scratch.thy (~/)
theory Scratch
imports
Complex_Main
"HOL-Analysis.Analysis"
begin
find_theorems "Inf _"
end
Purge
Continuous checking
Prover: ready
HOL
Starlike
Continuous_Extension
Multivariate_Analysis
Path_Connected
Arcwise_Connected
Homotopy
Homeomorphism
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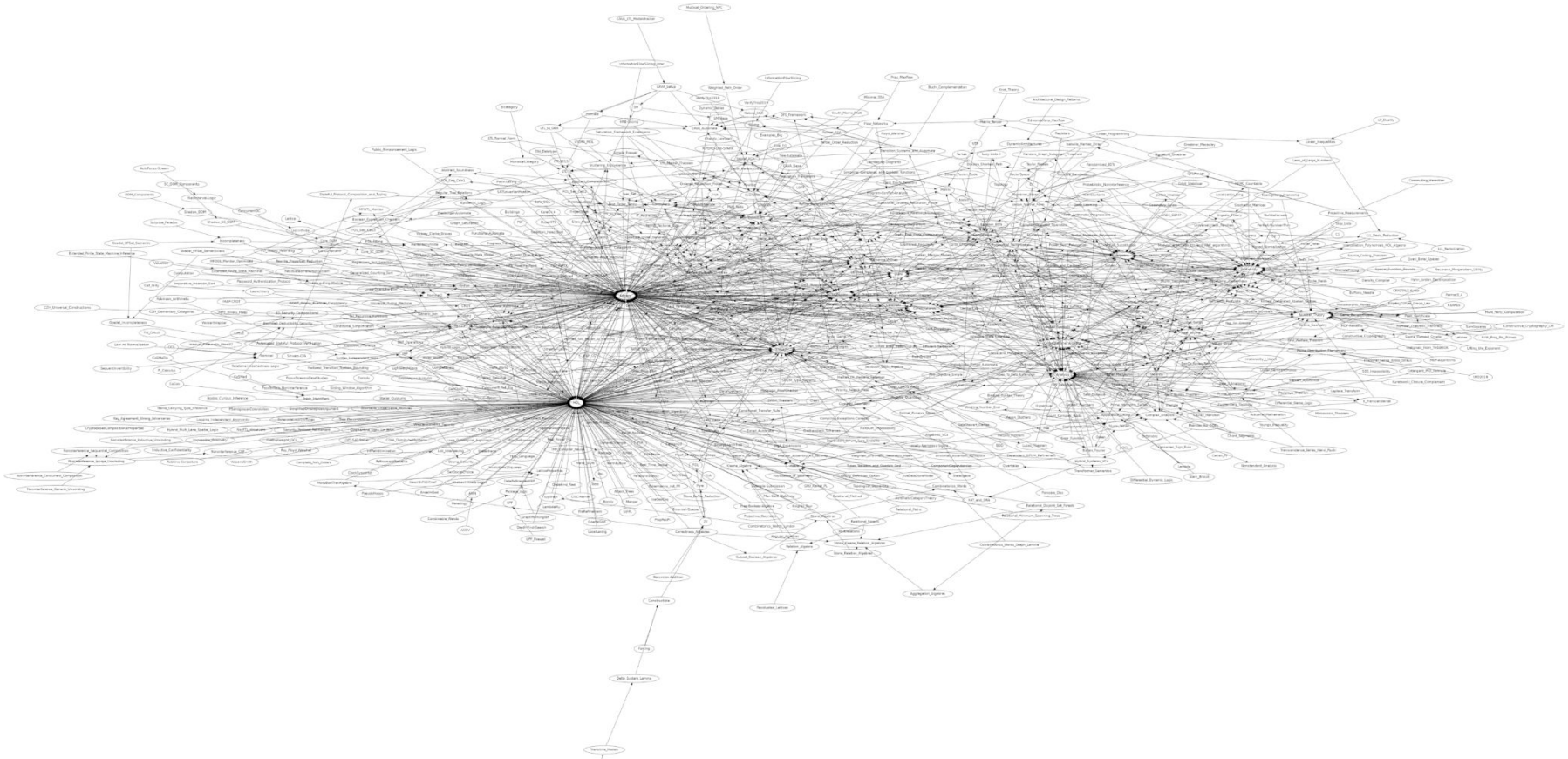
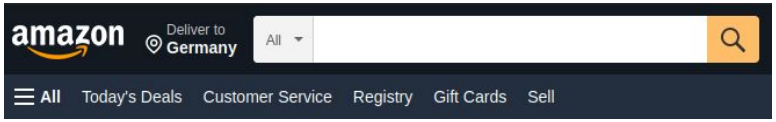
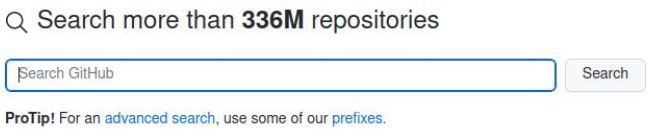
Locally
Abstract_Euclidean_Space
Polytope
Weierstrass_Theorems
Further_Topology
Jordan_Curve
Analysis
Scratch
Linter
Proof state
Auto update
Update
Search:
100%
find_theorems
"Inf _"
found 722 theorem(s) (40 displayed):
Enum.finite_lattice_class.bot_finite_def: bot = Inf UNIV
Complete_Lattices.complete_lattice_class.Inf_empty: Inf {} = top
Complete_Lattices.complete_lattice_class.Inf_UNIV: Inf UNIV = bot
Output
Query
Symbols
7,21 (97/103)
(isabelle,isabelle,UTF-8-Isabelle)
JVM: 2865/3244MB
ML: 7679/10712MB
12:22 AM

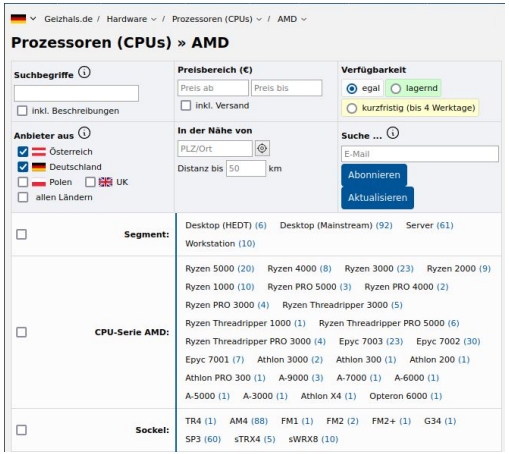
Hundreds of developments: The Archive of Formal Proofs.



But search in massive datasets exists!



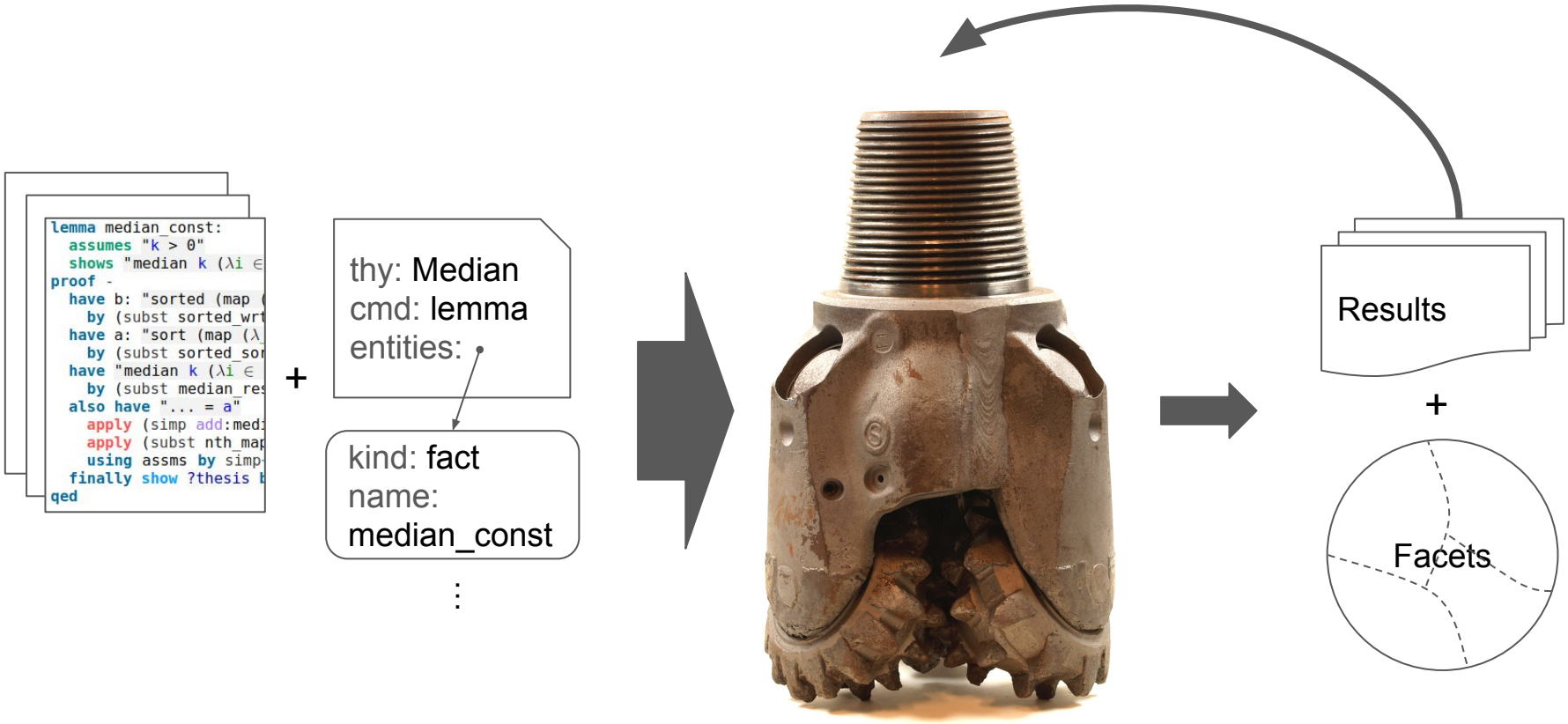




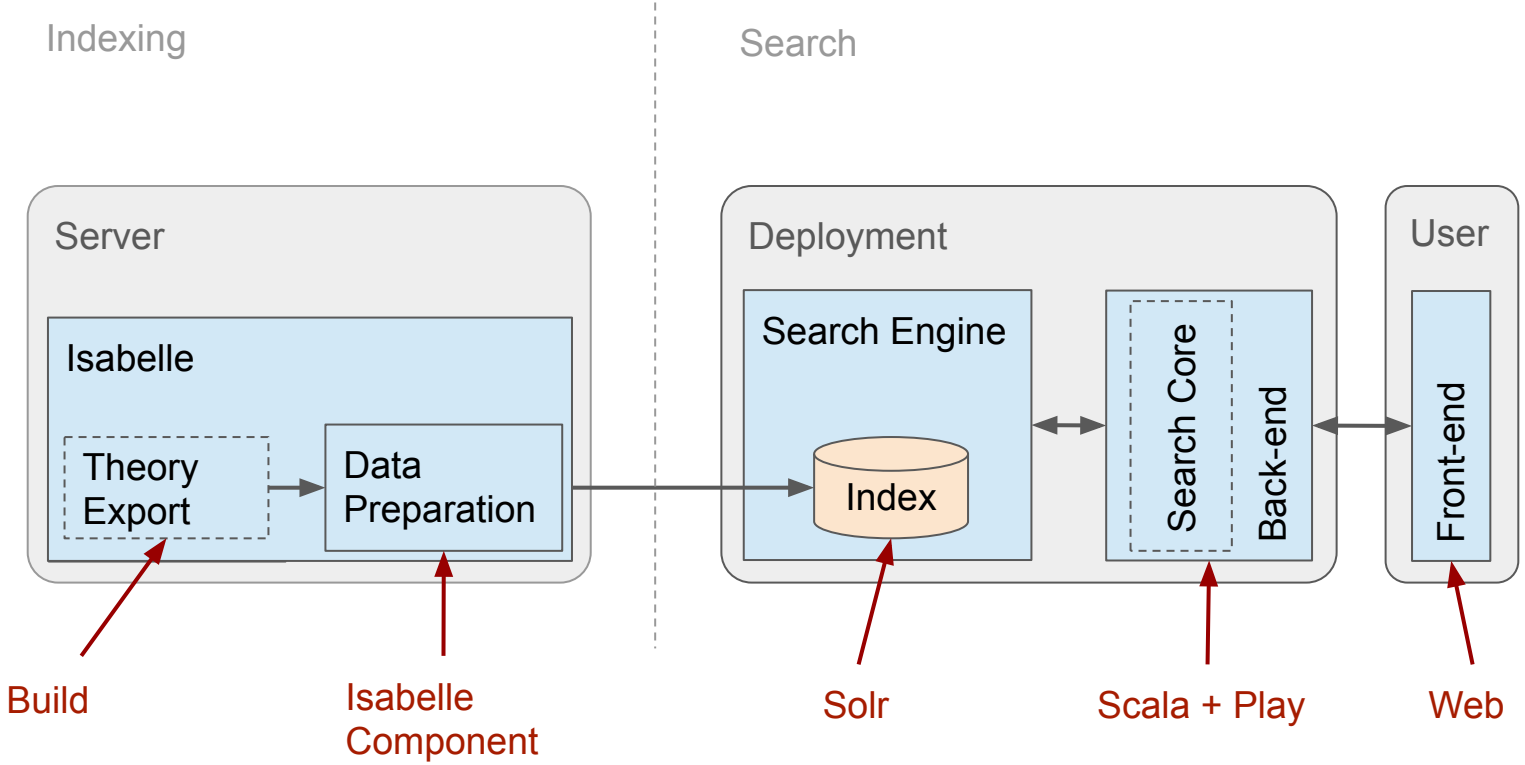
Search engines in the Isabelle landscape…

FindFacts: A play in three acts

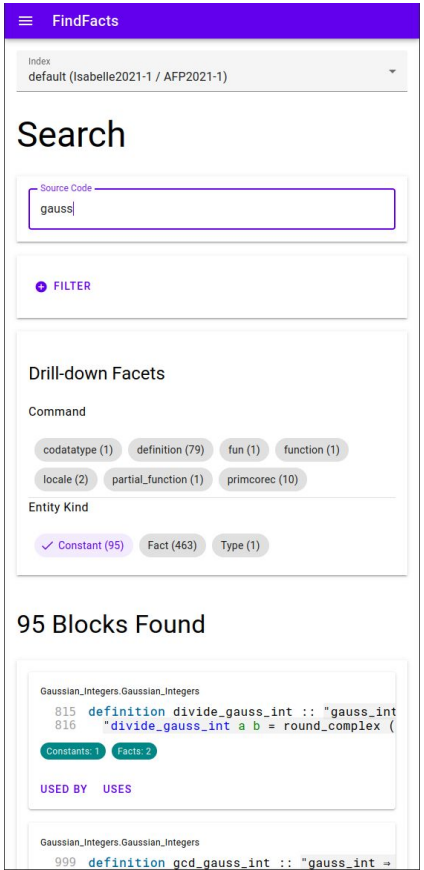
Act I. To find a fact

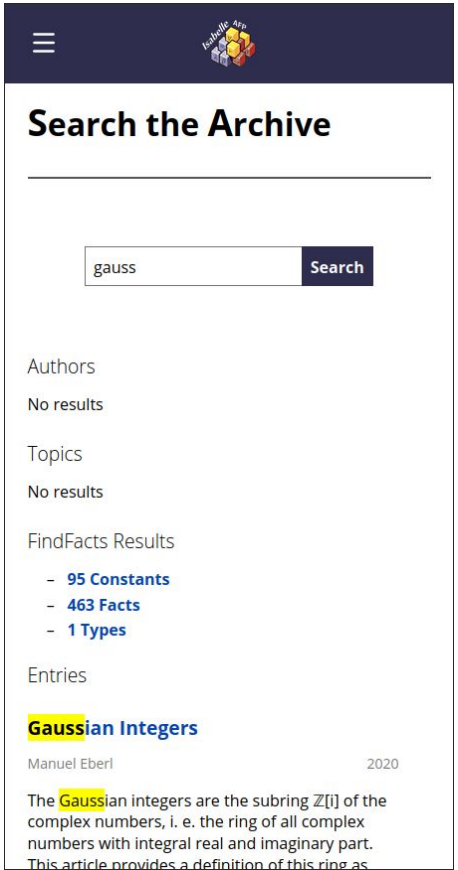


Act II. Revenge of the dump



Act III. Integration and salvation





Demo

https://search.isabelle.in.tum.de

Search

Index
default (Isabelle2021-1 / A

Source Code
Enter search terms with * wildcards...

FILTER

Search

Index
default (Isabelle2021-1 / A

Source Code
prime

FILTER

Drill-down Facets

Entity Kind — Constant (155) | Fact (1798) | Type (5)

2467 Blocks Found

Prime_Distribution_Elementary.Prime_Distribution_Elementary_Library

```
25 lemma smallest_prime_beyond_eval:
26   "prime n ⟹ smallest_prime_beyond n = n"
27   "¬prime n ⟹ smallest_prime_beyond n = smallest_prime_beyond (Suc n)"
28 proof -
29   assume "prime n"
30   thus "smallest_prime_beyond n = n"
31     by (rule smallest_prime_beyond_eq) auto
32 next
33   assume "¬prime n"
34   show "smallest_prime_beyond n = smallest_prime_beyond (Suc n)"
35   proof (rule antisym)
36     show "smallest_prime_beyond n ≤ smallest_prime_beyond (Suc n)"
37       by (rule smallest_prime_beyond_smallest)
38          (auto intro: order.trans[OF _ smallest_prime_beyond_le])
39   next
40     have "smallest_prime_beyond n ≠ n"
41       using prime_smallest_prime_beyond[of n] ‹¬prime n› by metis
```

Search

Index
default (Isabelle2021-1 / A

Source Code
prime

FILTER

Drill-down Facets

Entity Kind ✓ Constant (155) Fact (1798) Type (5)

155 Blocks Found

Dirichlet_Series.Dirichlet_Product

```
518 locale multiplicative_dirichlet_prod' =
519   f: multiplicative_function' f f_prime_power f_prime +
520   g: multiplicative_function' g g_prime_power g_prime
521   for f g :: "nat ⇒ 'a :: comm_semiring_1" and f_prime_power g_prime_power f_prime g_prime
522 begin
```

Constants: 1 Facts: 4

USED BY USES

Dirichlet_Series.Multiplicative_Function

```
71 locale multiplicative_function' = multiplicative_function f for f :: "nat ⇒ 'a :: comm_semiring_1" +
72   fixes f_prime_power :: "nat ⇒ nat ⇒ 'a" and f_prime :: "nat ⇒ 'a"
```

Search

Index
default (Isabelle2021-1 / A

Source Code
prime

Isabelle Command
Constant Type
Semantic Entity Name
Session
Source Code
Source Theory

✓ Constant (155) Fact (1798) Type (5)

155 Blocks Found

Dirichlet_Series.Dirichlet_Product

```
518 locale multiplicative_dirichlet_prod' =
519   f: multiplicative_function' f f_prime_power f_prime +
520   g: multiplicative_function' g g_prime_power g_prime
521   for f g :: "nat ⇒ 'a :: comm_semiring_1" and f_prime_power g_prime_power f_prime g_prime
522 begin
```

Constants: 1 Facts: 4

USED BY USES

Dirichlet_Series.Multiplicative_Function

```
71 locale multiplicative_function' = multiplicative_function f for f :: "nat ⇒ 'a :: comm_semiring_1" +
72   fixes f_prime_power :: "nat ⇒ nat ⇒ 'a" and f_prime :: "nat ⇒ 'a"
```

Search

Index
default (Isabelle2021-1 / A

Source Code
prime

Session
Enter phrase to filter for...

FILTER

Drill-down Facets

Entity Kind ✓ Constant (155) Fact (1798) Type (5)

155 Blocks Found

Dirichlet_Series.Dirichlet_Product

```
518 locale multiplicative_dirichlet_prod' =
519   f: multiplicative_function' f f_prime_power f_prime +
520   g: multiplicative_function' g g_prime_power g_prime
521   for f g :: "nat ⇒ 'a :: comm_semiring_1" and f_prime_power g_prime_power f_prime g_prime
522 begin
```

Constants: 1 Facts: 4

USED BY USES

Search

Index
default (Isabelle2021-1 / A

Source Code
prime

Session ONE OF HOL

FILTER

Drill-down Facets

Command	abbreviation (2) class (1) corec (2) definition (23) lift_definition (1) locale (6) qualified (2)
Entity Kind	✓ Constant (37) Fact (543)
Session	Auto2_HOL (1) HOL-Algebra (6) HOL-Computational_Algebra (12) HOL-Corec_Examples (2) HOL-Nonstandard_Analysis-Examples (1) HOL-Number_Theory (11) HOL-Proofs-Extraction (1) HOL-SMT_Examples (2) HOL-ex (1)

37 Blocks Found

HOL-Corec_Examples.Small_Concrete

```
35 corec prime_numbers where
36   "prime_numbers known_primes =
37     (let next_prime = head (fold (%n s. remove_multiples n s) known_primes (tail (tail all_numbers)))
38       S next_prime (prime_numbers (next_prime # known_primes)))"
```

Search

Index
default (Isabelle2021-1 / A

Source Code
prime

Session ONE OF HOL

FILTER

Drill-down Facets

Command	abbreviation (2) · class (1) · corec (2) · ✓ definition (23) · lift_definition (1) · locale (6) · qualified (2)
Entity Kind	✓ Constant (23) · Fact (23)
Session	Auto2_HOL (1) · HOL-Algebra (2) · HOL-Computational_Algebra (7) · HOL-Nonstandard_Analysis-Examples (1) · HOL-Number_Theory (8) · HOL-Proofs-Extraction (1) · HOL-SMT_Examples (2) · HOL-ex (1)

23 Blocks Found

HOL-Computational_Algebra.Factorial_Ring

```
491 definition prime :: "'a ⇒ bool" where
492   "prime p ↔ prime_elem p ∧ normalize p = p"
```

Constants: 2 Facts: 4

28 Blocks Found

HOL-Computational_Algebra.Factorial_Ring

```
491 definition prime :: "'a ⇒ bool" where
492   "prime p ↔ prime_elem p ∧ normalize p = p"
```

Constants: 2 Facts: 4

USED BY USES

HOL-Number_Theory.Eratosthenes

```
382 definition smallest_prime_beyond_aux :: "nat ⇒ nat ⇒ nat"
383 where
384   "smallest_prime_beyond_aux k n = smallest_prime_beyond n"
```

Constants: 1 Facts: 1

USED BY USES

HOL-Proofs-Extraction.Euclid

```
125 definition all_prime :: "nat list ⇒ bool"
126   where "all_prime ps ↔ (∀p∈set ps. prime p)"
```

Constants: 1 Facts: 1

USED BY USES

HOL-Algebra.Ring_Divisibility

```
 53 definition ring_prime :: "('a, 'b) ring_scheme ⇒ 'a ⇒ bool" ("ring'_prime⇘_⇙")
 54   where "ring_prime⇘R⇙ a ↔ (a ≠ 𝟬⇘R⇙) ∧ (prime R a)"
```

Constants: 1 Facts: 1

USED BY USES

HOL-Number_Theory.Eratosthenes

```
345 definition smallest_prime_between :: "nat ⇒ nat ⇒ nat option"
346 where
347   "smallest_prime_between m n =
348     (if (∃p. prime p ∧ m ≤ p ∧ p ≤ n) then Some (smallest_prime_beyond m) else None)"
```

Search

Index
default (Isabelle2021-1 / A

Source Code

Uses
```
definition prime :: "'a ⇒ bool" where
  "prime p ↔ prime_elem p ∧ normalize p = p"
```

FILTER

Drill-down Facets

Entity Kind Constant (56) Fact (855)

856 Blocks Found

HOL-Computational_Algebra.Factorial_Ring
```
494 lemma not_prime_0 [simp]: "¬prime 0" by (simp add: prime_def)
```
Facts: 2

USED BY USES

HOL-Computational_Algebra.Factorial_Ring
```
496 lemma not_prime_unit: "is_unit x ⟹ ¬prime x"
497   using prime_elem_not_unit[of x] by (auto simp add: prime_def)
```

What do people search for? 16K Queries:

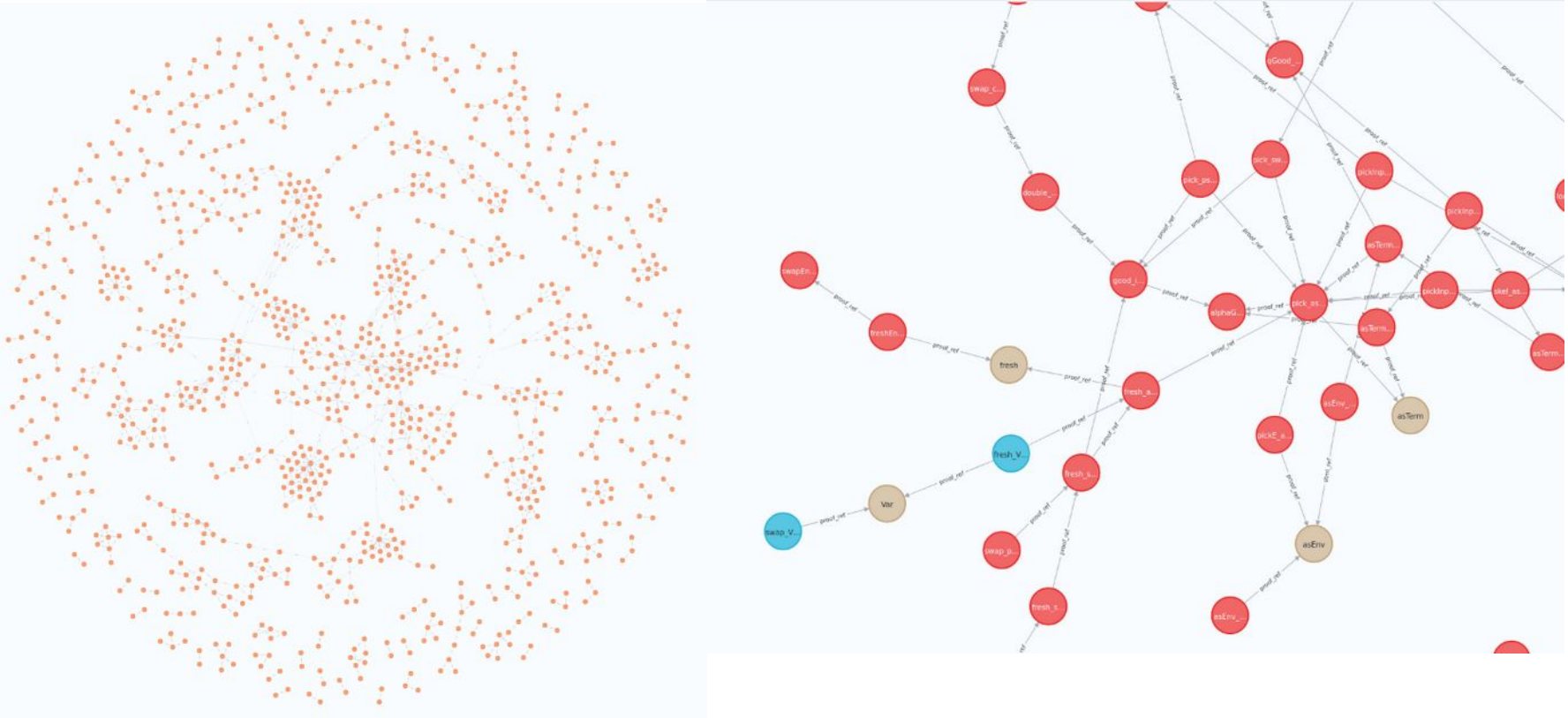
“prime”	(834)
“*”	(291)
“ring”	(144)
“comm*”	(129)
“pigeonhole”	(118)
“matrix”	(116)
“∩”	(116)

SErAPIS

Supported by the ERC Advanced Grant ALEXANDRIA, Project 742178

https://www.cl.cam.ac.uk/~lp15/Grants/Alexandria/

The Isabelle Libraries



Archive of Formal Proofs: 3,396,200 lines of code (as of April, 2022)

The Isabelle Libraries and AFP

- Theories are formed using many (interconnected) artefacts
  - Theorems, lemmata, corollaries, definitions, axiomatisations.
  - Locales, sublocales, classes and subclasses + interpretations, instantiations and abbreviations.

- Potentially daunting for new Isabelle users

“I’m looking for Harmonic (numbers), where do I even start?”

1. Novice users might have an idea of what is needed to complete proof.

   BUT not enough experience with library organisation and naming conventions to find what they need.

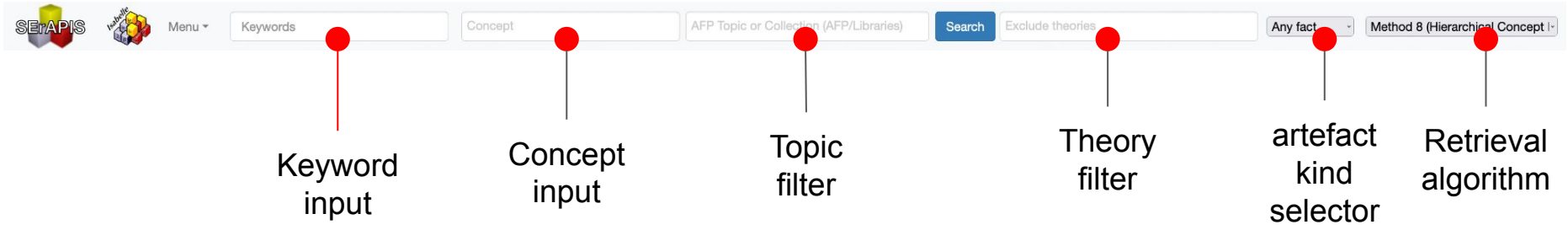
2. Modern search users expect an experience akin to a google search box.

   e.g., input a “bag-of-words” search in a search box

The SErAPIS Search Engine

- **SErAPIS**: **S**earch **E**ngine by the **A**lexandria **P**roject for **IS**abelle
- Designed to help new Isabelle users navigate the Libraries and AFP
    - A *Concept-oriented* (NL) search engine.
    - Aims to minimise user input with “intelligent” retrieval algorithms doing the work
- Designed to facilitate research into Isabelle retrieval
    1. Replaceable components.
    2. Index and front-end support multiple retrieval models.
    3. Anonymised session and query tracking.
    4. Supports relevance feedback directly in the UI.

A Tool for New Isablelle Users

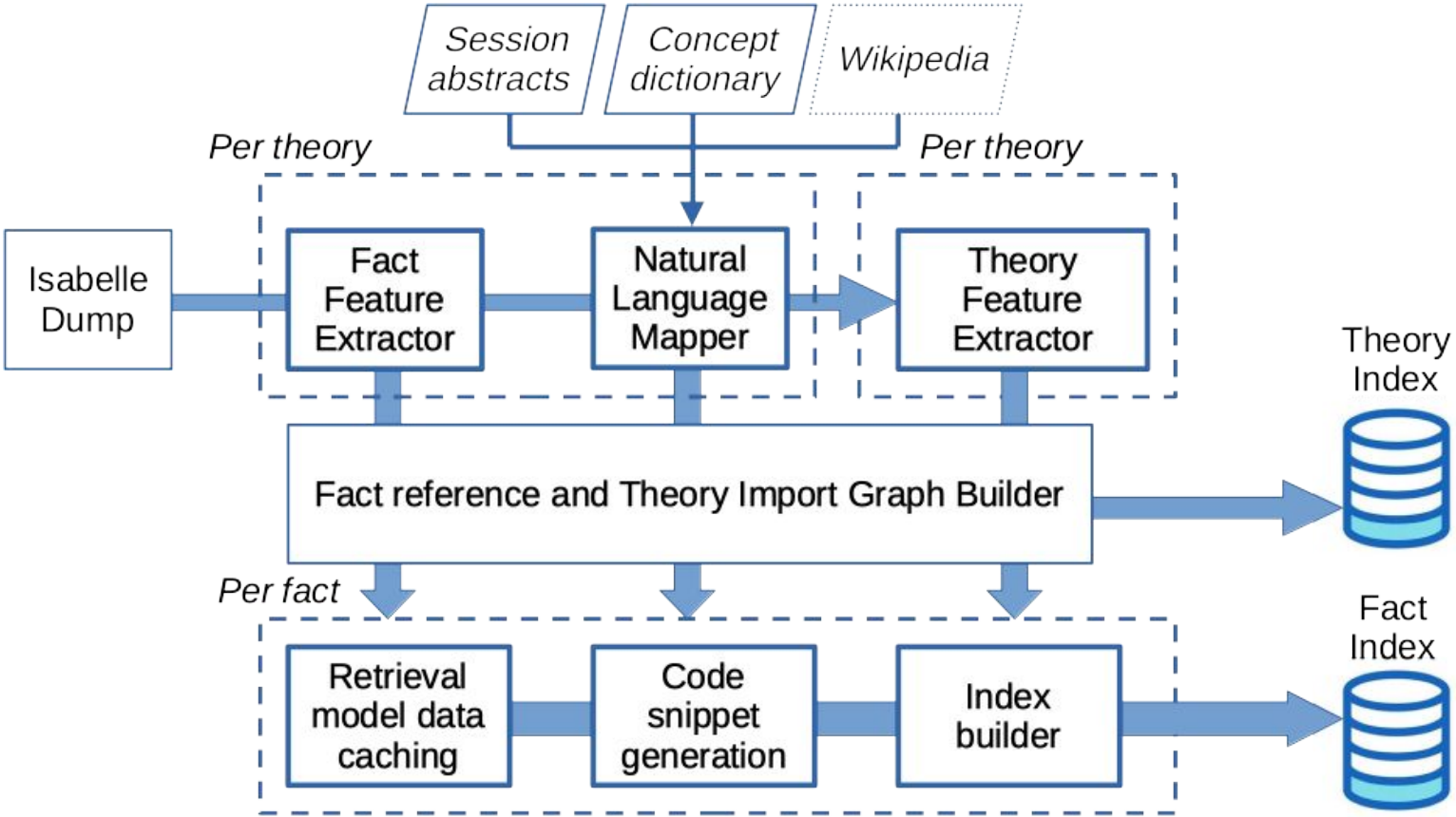


- What are “concepts”?
  - words or phrases that refer to mathematical ideas (e.g., objects and structures)
  - most concepts are nouns or noun phrases pre-modified by adjectives.

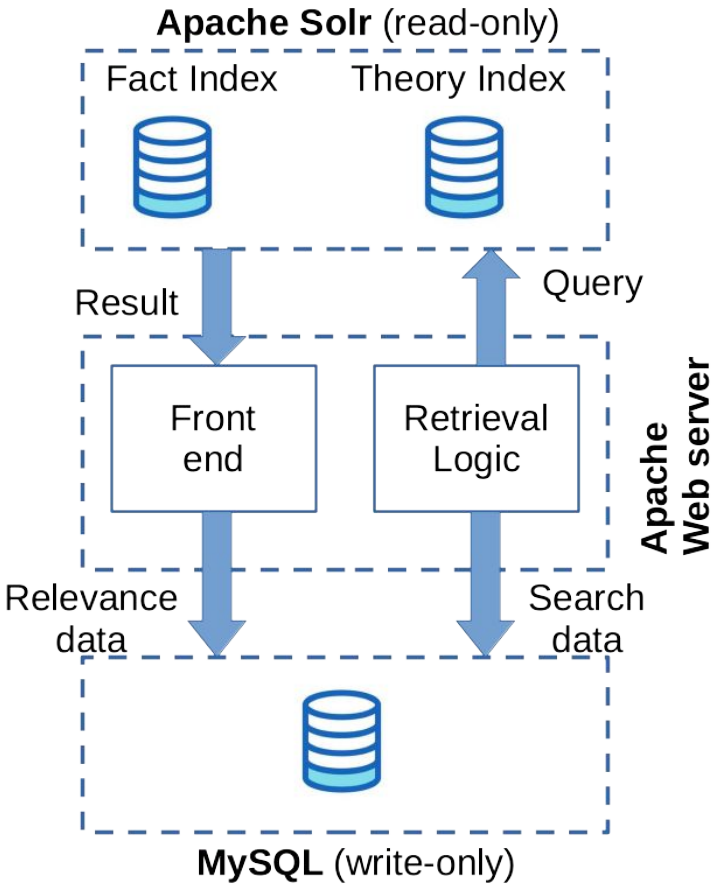
> *Let P be a* ***parabolic subgroup*** *of $GL(n)$ with* ***Levi decomposition*** *$P = MN$, where N is the* ***unipotent radical***. *Let π be an* ***irreducible representation*** *of $M(\mathbb{Z}_p)$ inflated to $P(\mathbb{Z})$.*

- What do we mean by *concept-oriented*?
  - “understand” the mathematical concepts/ideas behind a search.
  - Associate closely related notions.

Architecture to Support Research - I



Architecture to Support Research - II

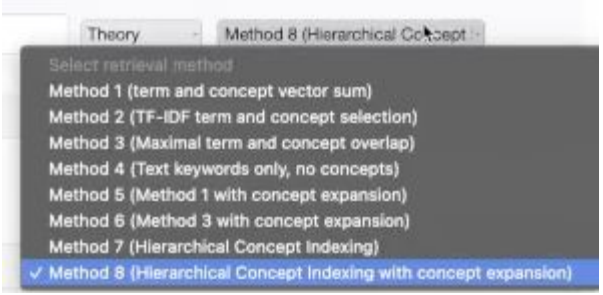


- Index can expose multiple fields for models

	Feature	Kind	Description
1	name	String	The name of the fact
2	kind	String	The kind of the fact: theorem, lemma, definition or axiom.
3	theory_key	String	Identifier for the source theory in Library_Theory format.
4	theory_name	String	The name of the source theory, produced from its filename.
5	comments	Text	Comments above the fact in the theory file.
6	incomments	Text	Comments appearing inside the fact's body.

13	proofblocks	Integer	The number of "proof" blocks in the fact's body.
14	byblocks	Integer	The number of "by" blocks in the fact's body.
15	proof_commandvec	Vector	Commands used in the proof block of the fact and the frequency of their evokation.

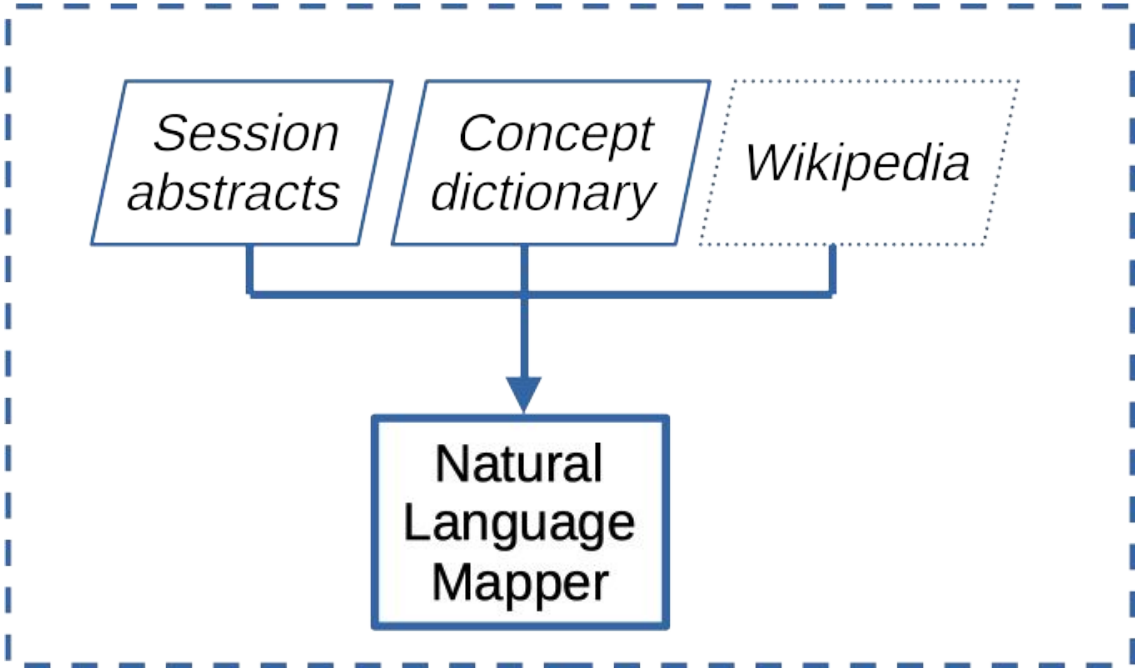
- Front end can interface to multiple models



Architecture to Support Research - III

- User-provided relevance judgements
- Clickthrough data
- Sessions: Evolution of queries

A Peek Inside the Box



Current Implementation

1. Concept Index for Wikipedia Mathematics articles
2. Fact features + Abstracts for each fact.
3. Top 20 most relevant Wiki articles for each fact.

Mapping Concepts to Facts - I

Cauchy_Schwarz_ineq
(HOL-Analysis/Inner_Product.thy)

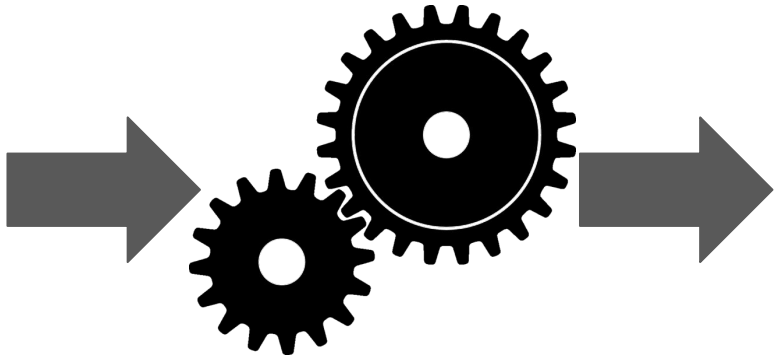
Rank	Title
1	Cauchy–Schwarz inequality
2	Augustin-Louis Cauchy
3	Cauchy–Riemann equations
4	Cauchy sequence
5	Schwarz list
6	Cauchy momentum equation
7	Cauchy–Kowalevski theorem
8	Cauchy surface
9	Cauchy product
10	Albert Schwarz
11	Schwarz lemma
12	Binet–Cauchy identity
13	Cauchy theorem (group theory)
14	Cauchy–Rassias stability
15	Schwarz reflection principle
16	Schwarz–Ahlfors–Pick theorem
17	Abstract additive Schwarz method
18	Schwarz minimal surface
19	Schwarz triangle function
20	Cauchy theorem

meet_dual
(HOL-Algebra/Lattice.thy)

Rank	Title
1	Join and meet
2	Langlands dual group
3	Petrie dual
4	Lattice (order)
5	De Groot dual
6	Reductive dual pair
7	Complete lattice
8	Heyting algebra
9	Free lattice
10	F-algebra
11	Boolean algebra (structure)
12	Capelli identity
13	Skew lattice
14	Closure operator
15	0,1-simple lattice
16	Comparison of topologies
17	Fixed-point combinator
18	Distributive lattice
19	Semimodular lattice
20	Birkhoff representation theorem

Mapping Concepts to Facts - II

Rank	Title
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	



Term Vector

Term vector for the field: **conceptvec**

Term	Freq.
completeness theorem	20
deductive system	19
formula	18
logic	13
theory	13
proof	10
theorem	9
compactness theorem	8
language	8
completeness	7
model	7

OK

Top 20 Wikipedia Math Articles

Decision/Selector Function

Distributional representation

Demo

https://behemoth.cl.cam.ac.uk/search/

https://behemoth.cl.cam.ac.uk/search/SErAPIS_online_user_guide.pdf

SErAPIS Menu Keywords harmonic number × AFP Topic or Collection (AFP/Libraries) Search HOL-Analysis.Complete_Measure × Any fact Method 8 (Hierarchical Concept I

Will you provide us with relevance feedback for this query?

Please indicate whether you will be providing relevance feedback on the **first 10 results** for this query by clicking on the button.

I WILL Help!

1 **harm** definition (Libraries) HOL-Analysis.Harmonic_Numbers

Used by

Preview snippet

```
definition‹tag important› harm :: "nat ⇒ 'a :: real_normed_field" where
"harm n = (∑k=1..n. inverse (of_nat k))"
```

2 **harm_expand** lemma (Libraries) HOL-Analysis.Harmonic_Numbers

Used by

Uses

Preview snippet

```
lemma harm_expand:
"harm 0 = 0"
"harm (Suc 0) = 1"
"harm (numeral n) = harm (pred_numeral n) + inverse (numeral n)"
proof -
have "numeral n = Suc (pred_numeral n)" by simp
also have "harm … = harm (pred_numeral n) + inverse (numeral n)"
by (subst harm_Suc, subst numeral_eq_Suc[symmetric]) simp
finally show "harm (numeral n) = harm (pred_numeral n) + inverse (numeral n)" .
qed (simp_all add: harm_def)
```

3 **harm_pos** lemma (Libraries) HOL-Analysis.Harmonic_Numbers

Used by

Uses

Preview snippet

SErAPIS Menu Keywords asymptotic density × AFP Topic or Collection (AFP/Libraries) Search Exclude theories Any fact Method 8 (Hierarchical Concept

Will you provide us with relevance feedback for this query?

1 **lower_asymptotic_density_intersection** lemma [Mathematics/Probability_theory] (AFP) Ergodic_Theory.Asymptotic_Density

Uses

HOL-Analysis.Cross3.algebra_simps HOL-Library.FSet.Diff_Int

Preview snippet

```
lemma lower_asymptotic_density_intersection:
"lower_asymptotic_density A + lower_asymptotic_density B ≤ lower_asymptotic_density (A ∩ B) + 1"
using upper_asymptotic_density_union[of "UNIV - A" "UNIV - B"]
unfolding lower_upper_asymptotic_density_complement by (auto simp add: algebra_simps Diff_Int)
```

2 **upper_asymptotic_density_in_01** lemma [Mathematics/Probability_theory] (AFP) Ergodic_Theory.Asymptotic_Density

Used by

Ergodic_Theory.Asymptotic_Density.upper_asymptotic_density_infinite_interval Ergodic_Theory.Asymptotic_Density.upper_asymptotic_densityI

Preview snippet

3 **upper_asymptotic_density_lim** lemma [Mathematics/Probability_theory] (AFP) Ergodic_Theory.Asymptotic_Density

Preview snippet

4 **upper_asymptotic_density_subset** lemma [Mathematics/Probability_theory] (AFP) Ergodic_Theory.Asymptotic_Density

Uses

HOL.Set_Interval.lessThan HOL.Set_Interval.finite_lessThan ZF.equalities.Int_lower2 HOL-Decision_Procs.Dense_Linear_Order.not_le LTL_to_DRA.Semi_Mojmir.card_mono Transition_Systems_and_Automata.Basic.finite_subset HOL.Lattices.left_idem HOL.Filter.eventually_mono HOL-Library.Float.order_trans LinearQuantifierElim.FRE.dense HOL-Library.Countable_Set_Type.Int_mono HOL.Nat.of_nat_le_iff HOL-Matrix_LP.SparseMatrix.less_imp_le

Preview snippet

```
lemma upper_asymptotic_density_subset:
```

Will you provide us with relevance feedback for this query?

1 **ramsey1** lemma (Libraries) HOL-Library.Ramsey

Uses

Preview snippet

```
lemma ramsey1: "∃N::nat. partn_lst {..<N} [q0,q1] 1"
proof -
have "∃i<Suc (Suc 0). ∃H∈nsets {..<Suc (q0 + q1)} ([q0, q1] ! i). f ` nsets H (Suc 0) ⊆ {i}"
if "f ∈ nsets {..<Suc (q0 + q1)} (Suc 0) ⟶ {..<Suc (Suc 0)}" for f
proof -
define A where "A ≡ λi. {q. q ≤ q0+q1 ∧ f {q} = i}"
have "A 0 ∪ A 1 = {..q0 + q1}"
using that by (auto simp: A_def PiE_iff nsets_one lessThan_Suc_atMost le_Suc_eq)
moreover have "A 0 ∩ A 1 = {}"
by (auto simp: A_def)
ultimately have "q0 + q1 ≤ card (A 0) + card (A 1)"
by (metis card_Un_le card_atMost eq_imp_le le_Suc le_trans)
then consider "card (A 0) ≥ q0" | "card (A 1) ≥ q1"
by linarith
then obtain i where "i < Suc (Suc 0)" "card (A i) ≥ [q0, q1] ! i"
by (metis One_nat_def less le nth_Cons_0 nth_Cons_Suc zero_less_Suc)
then obtain B where "B ⊆ A i" "card B = [q0, q1] ! i" "finite B"
by (meson obtain_subset_with_card_n)
then have "B ∈ nsets {..<Suc (q0 + q1)} ([q0, q1] ! i) ∧ f ` nsets B (Suc 0) ⊆ {i}"
by (auto simp: A_def nsets_def card_1_singleton_iff)
then show ?thesis
using ‹i < Suc (Suc 0)› by auto
qed
then show ?thesis
by (clarsimp simp: partn_lst_def) blast
qed


subsubsection ‹Ramsey's theorem with two colours and arbitrary exponents (hypergraph version)›
```

2 **pigeonhole_principle** lemma [Computer_science/Data_structures] (AFP) List-Infinite.SetInterval2

Preview snippet

Will you provide us with relevance feedback for this query?

Please indicate whether you will be providing relevance feedback on the **first 10 results** for this query by clicking on the button.

I WILL Help!

1 **summable_pre_sin** lemma (Libraries) HOL-Real_Asymp.Multiseries_Expansion

Uses

Preview snippet

2 **cos_conv_pre_cos** lemma (Libraries) HOL-Real_Asymp.Multiseries_Expansion

Uses

HOL.Parity.evenE HOL.Series.sums_iff HOL.Series.sums_mono_reindex HOL-Computational_Algebra.Formal_Laurent_Series.fun_eq_iff HOL.Orderings.strict_mono HOL-Real_Asymp.Multiseries_Expansion.mssnth_cos_series_stream HOL.Power.power_mult HOL.Transcendental.cos_coeff HOL-Real_Asymp.Multiseries_Expansion.powser

Preview snippet

```
lemma cos_conv_pre_cos:
"cos x = powser (msllist_of_msstream cos_series_stream) (x ^ 2)"
proof -
have "(λn. cos_coeff (2 * n) * x ^ (2 * n)) sums cos x"
using cos_converges[of x]
by (subst sums_mono_reindex[of "λn. 2 * n"])
(auto simp: strict_mono_def cos_coeff_def elim!: evenE)
also have "(λn. cos_coeff (2 * n) * x ^ (2 * n)) =
(λn. mssnth cos_series_stream n * (x ^ 2) ^ n)"
by (simp add: fun_eq_iff mssnth_cos_series_stream cos_coeff_def power_mult)
finally have sums: "(λn. mssnth cos_series_stream n * x^2 ^ n) sums cos x" .
thus ?thesis by (simp add: powser_def sums_iff)
qed
```

3 **sin_conv_pre_sin** lemma (Libraries) HOL-Real_Asymp.Multiseries_Expansion

Uses

Preview snippet

Menu

Keywords

cauchy-schwarz inequality ×

AFP Topic or Collection (AFP/Libraries)

Search

Exclude theories

Theory

Method 8 (Hierarchical Concept

Will you provide us with relevance feedback for this query?

Please indicate whether you will be providing relevance feedback on the **first 10 results** for this query by clicking on the button.

I WILL Help!

1 **CauchySchwarz** Cauchy.CauchySchwarz Cauchy [Mathematics/Analysis] (AFP)

2 **No_Cloning** Isabelle_Marries_Dirac.No_Cloning Isabelle_Marries_Dirac [Computer_science/Algorithms/Quantum_computing Mathematics/Physics/Quantum_information] (AFP)

3 **CauchysMeanTheorem** Cauchy.CauchysMeanTheorem Cauchy [Mathematics/Analysis] (AFP)

4 **Example_Metric** HOL-Eisbach.Example_Metric HOL-Eisbach (Libraries)

5 **Equality** CTT.Equality CTT (Libraries)

6 **Wfd** CCL.Wfd CCL (Libraries)

7 **Imperative_Quicksort** HOL-Imperative_HOL.Imperative_Quicksort HOL-Imperative_HOL (Libraries)

8 **RG_Tran** HOL-Hoare_Parallel.RG_Tran HOL-Hoare_Parallel (Libraries)

9 **Correctness** IOA-ABP.Correctness IOA-ABP (Libraries)

10 **Metric_Arith_Examples** HOL-Analysis-ex.Metric_Arith_Examples HOL-Analysis-ex (Libraries)

11 **Big_Step_Value** CakeML_Codegen.Big_Step_Value CakeML_Codegen [Computer_science/Programming_languages/Compiling Logic/Rewriting] (AFP)

12 **Correctness2** JinjaDCI.Correctness2 JinjaDCI [Computer_science/Programming_languages/Language_definitions] (AFP)

13 **SC_Cut** Propositional_Proof_Systems.SC_Cut Propositional_Proof_Systems [Logic/Proof_theory] (AFP)

14 **Impl_RBT_Map** Collections.Impl_RBT_Map Collections [Computer_science/Data_structures] (AFP)

15 **BVSpecTypeSafe** HOL-MicroJava.BVSpecTypeSafe HOL-MicroJava (Libraries)

16 **Hash_Table** Separation_Logic_Imperative_HOL.Hash_Table Separation_Logic_Imperative_HOL [Computer_science/Programming_languages/Logics] (AFP)

17 **Comb** HOL-Induct.Comb HOL-Induct (Libraries)

18 **Ramsey** ZF-ex.Ramsey ZF-ex (Libraries)

19 **BVSpecTypeSafe** JinjaDCI.BVSpecTypeSafe JinjaDCI [Computer_science/Programming_languages/Language_definitions] (AFP)

20 **Forcing_Theorems** Forcing.Forcing_Theorems Forcing [Logic/Set_theory] (AFP)

Closing Remarks

- Ongoing and Future Work

  1. Migrate from Isabelle Dump to Isabelle-Scala and Isabelle build.
  2. Keep index up-to-date with Isabelle + AFP releases.
  3. Develop new NL mapping functions.
  4. Build a Test Collection for evaluating Isabelle NL search.

- Links

  - SErAPIS search engine: **https://behemoth.cl.cam.ac.uk/search/**
  - SErAPIS User Guide: **https://behemoth.cl.cam.ac.uk/search/SErAPIS_online_user_guide.pdf**
  - My website: https://www.cl.cam.ac.uk/~yas23/

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Questions?

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