Breaking the diamond in Superposition

Márton Hajdu, Laura Kovács, Michael Rawson, Andrei Voronkov

TECHNISCHE UNIVERSITÄT WIEN

Superposition calculus (ground)

$$\text{(Sup)}\ \frac{s[u] \bowtie t \lor C \qquad u \simeq w \lor D}{s[w] \bowtie t \lor C \lor D}$$

where

(1) $\bowtie \in \{\simeq, \not\simeq\}$,

(2) $s \succ t$ and $u \succ w$,

(3) $s \bowtie t \succ C$ and $u \simeq w \succ D$.

$$\text{(EqRes)}\ \frac{s \not\simeq s \lor C}{C}$$

where $s \not\simeq s \succ C$.

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Diamond property

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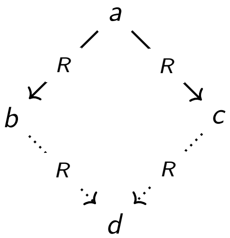
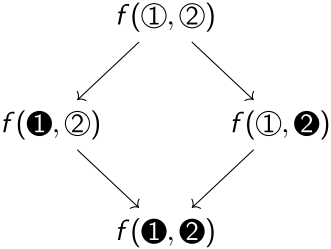


Rewriting independent/parallel positions

Given equations ⓘ $\simeq$ 🅘 for $1 \leq i \leq 3$



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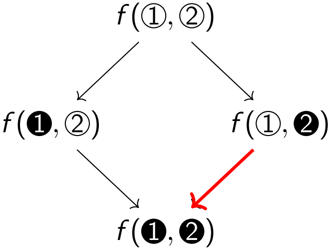

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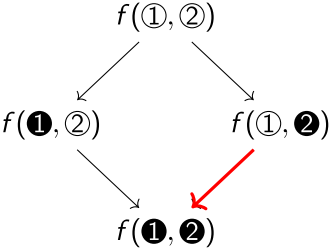

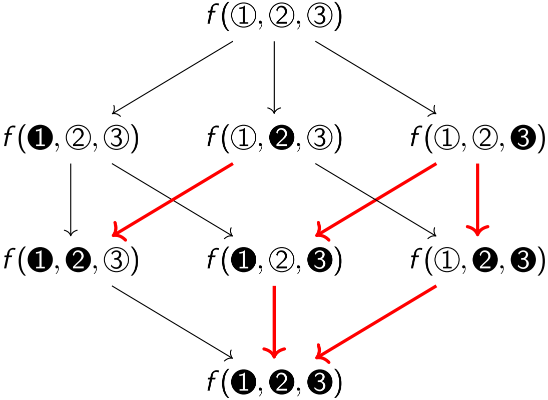

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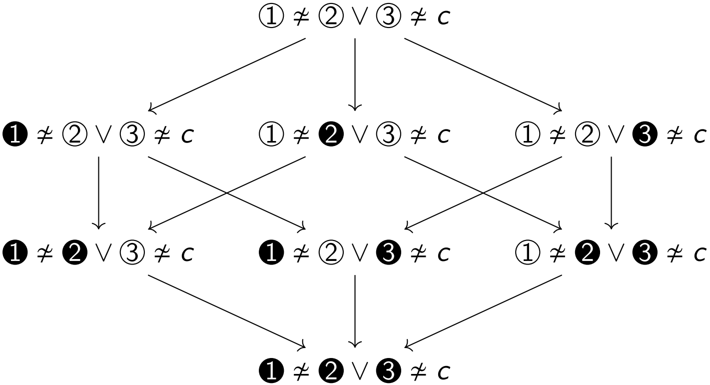
What about independent rewrites in superposition?

Given equations ⓘ $\simeq$ ➊ for $1 \leq i \leq 3$ and term order ① $\succ$ ② $\succ$ ③ $\succ$ ❶ $\succ$ ❷ $\succ$ ❸ $\succ$ c



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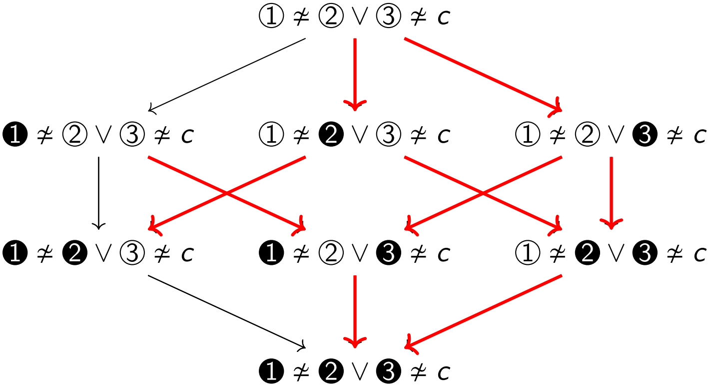

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Given ⓘ $\simeq$ ❶ for $1 \leq i \leq 3$, and $f(\cdot) \succ c$

$$f(①, f(②, ③)) \not\simeq c$$

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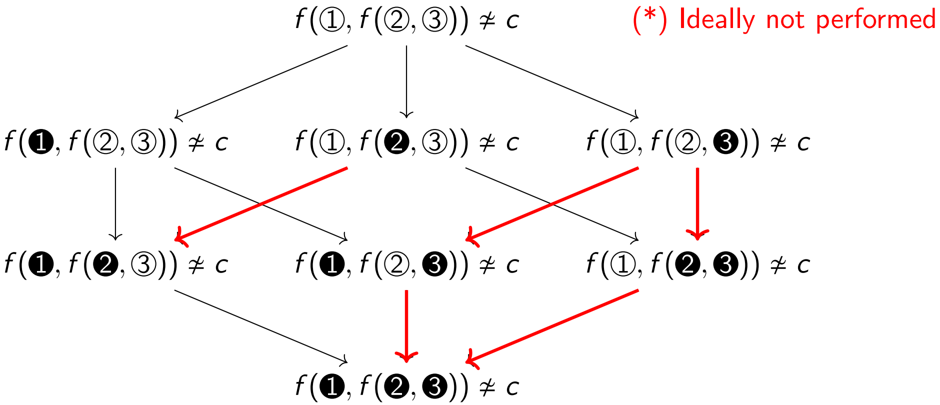

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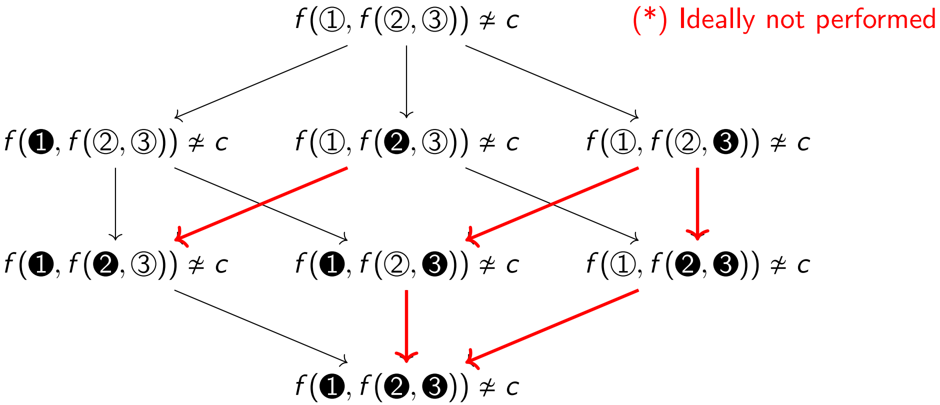

Subsumption deals with duplicates!

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- Can duplicates be efficiently avoided while remaining complete?

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- Can duplicates be efficiently avoided while remaining complete?
- **Idea**: rewrite a position only if no position to its right (or left) has been rewritten.

An example

Given ⓘ $\simeq$ ❶ for $1 \leq i \leq 3$ and $f(\cdot) \succ c$

$$f(①, f(②, ③))^{\langle \epsilon \rangle} \not\simeq c$$

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Superposition calculus (with left-to-right ordered independent rewrites)

$$\text{(Sup)}\ \frac{s[u]_r{}^{\langle p \rangle} \bowtie t \lor C \qquad u \simeq w^{\langle q \rangle} \lor D}{s[w]_r{}^{\langle rq \rangle} \bowtie t \lor C \lor D}$$

where

(1) $s \succ t$ and $u \succ w$,
(2) $s \bowtie t \succ C$ and $u \simeq w \succ D$,
(4) $p \nparallel r$ or $p <_{lex} r$.

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Experimental results

Running Vampire on TPTP problems with `-sa discount`:

Timeout	Master	New branch	Total
10s	10718 (59)	10716 (57)	25257
60s	11536 (81)	11524 (69)	

In general, it saturates the search space a bit faster.

Where is the new approach worse on some problems?

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t_4

t_1

$$f(①, f(❷, ③))^{\langle 2.1\rangle} \not\simeq c$$

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t_3

If the clauses get activated at timepoints $t_1 < t_2 < t_3 < t_4$, then if $t_1 \ll t_2$, there is a good chance that $t_3 \lll t_4$. Finding the same proof takes longer!

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- **Idea:** change the order from left-to-right ($\rightarrow$) to right-to-left ($\leftarrow$) when needed

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- If the clause with the rightmost rewrite is activated first, use $\leftarrow$

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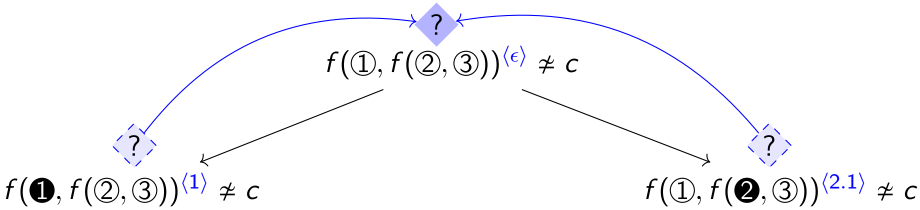

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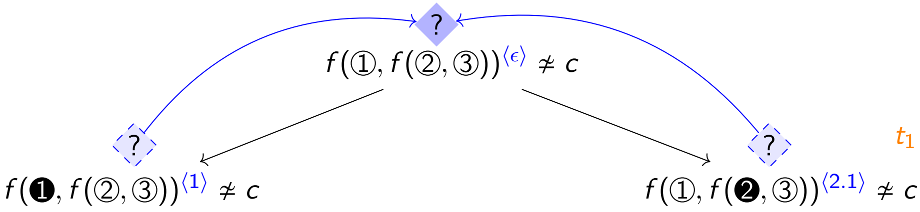

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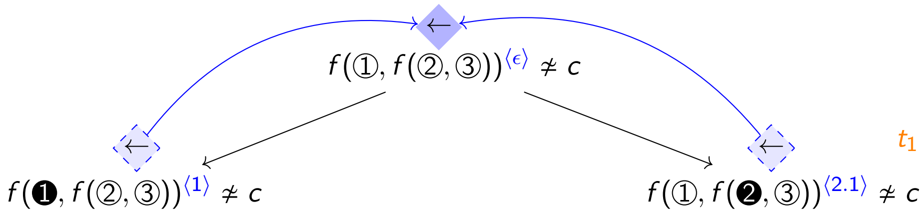
Given $\textcircled{i} \simeq \textcircled{\textbf{i}}$ for $1 \leq i \leq 3$ and $f(\cdot) \succ c$



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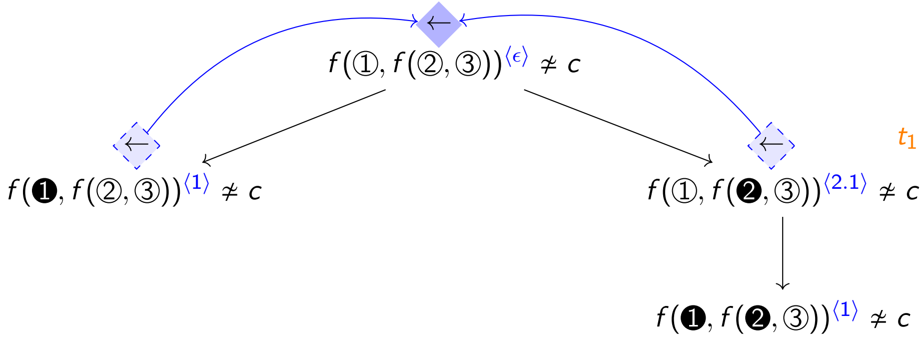

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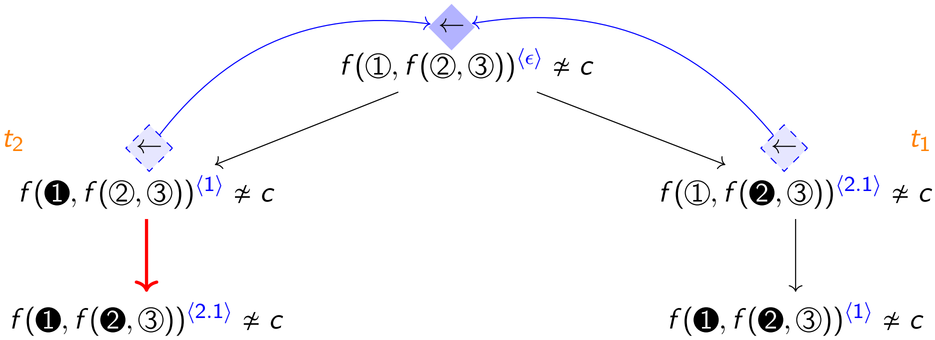

Future work & conclusion

- Still underwhelming results
- The number of diamonds a certain clause participates in is often huge
- It is possible to extend to arbitary permutations (but worth it?)
- Prove completeness
- Find benchmarks/strategies where this is useful