Two Learning Operators for Clause Selection Guidance

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Machine Learning Boosted Automated Theorem Proving

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ATP technology:

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What are the differences? What is the same? Which one is better?

Outline

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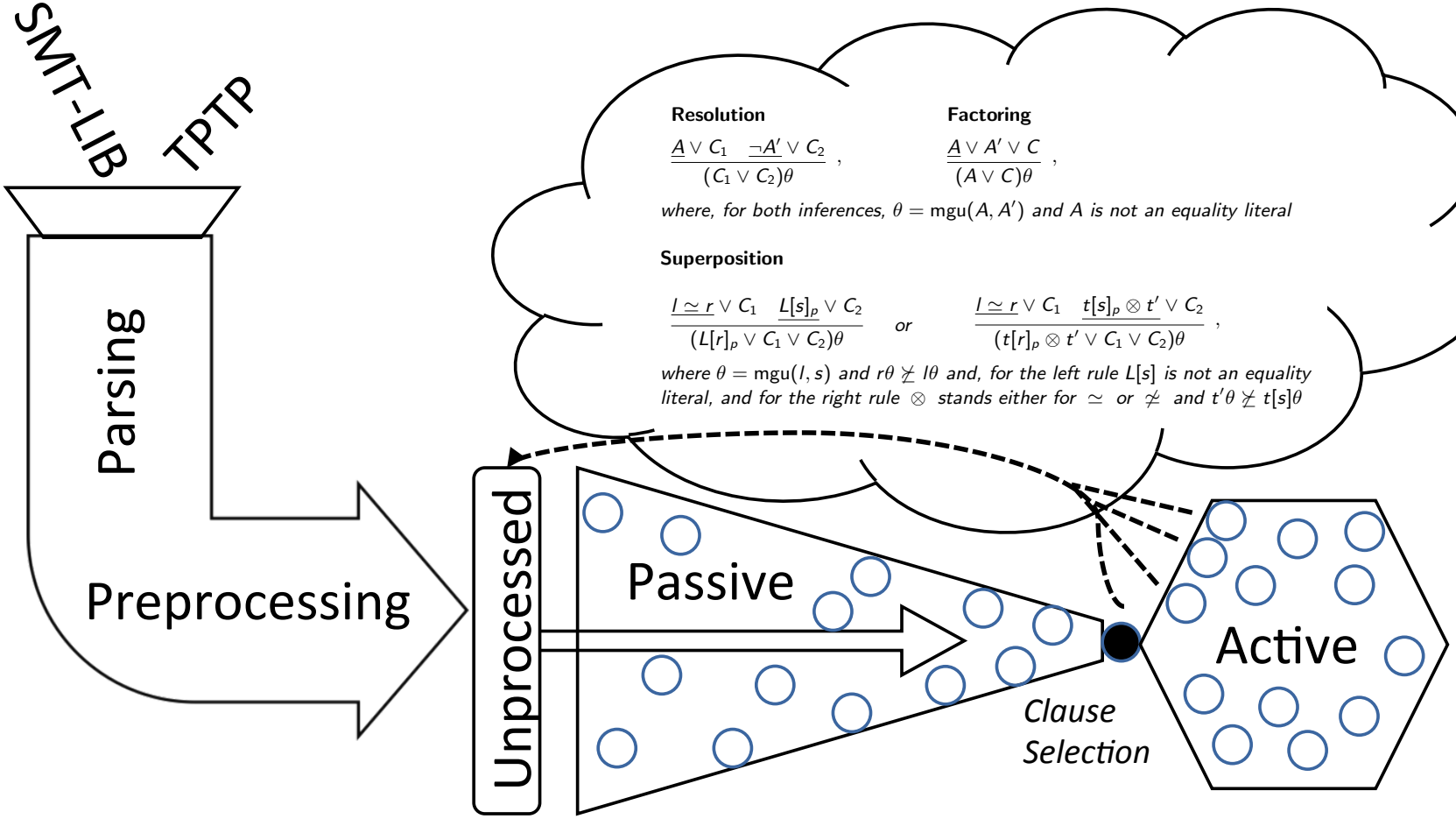

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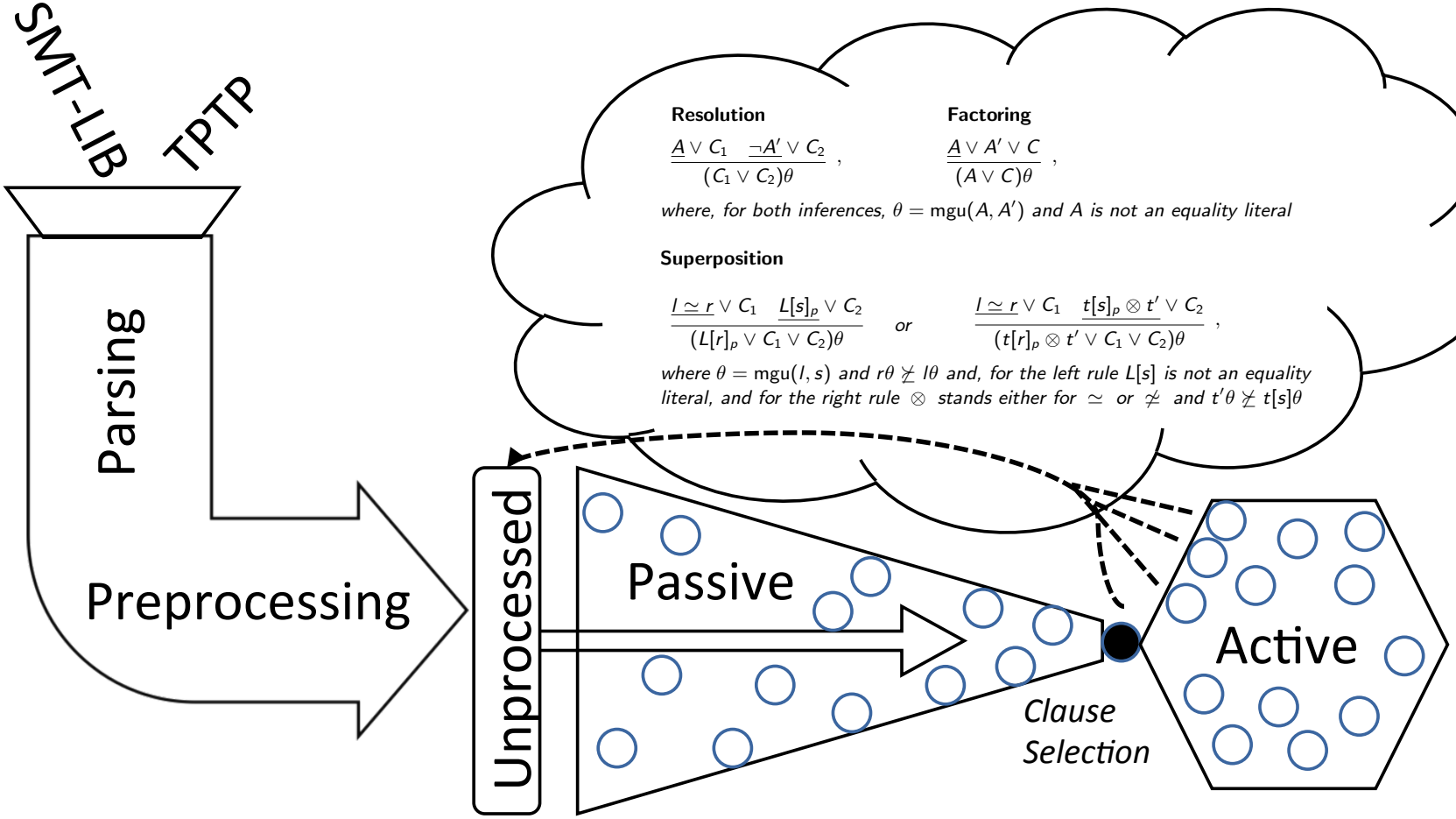

At a typical successful end: $|Passive| \gg |Active| \gg |Proof|$

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Take simple clause evaluation criteria:

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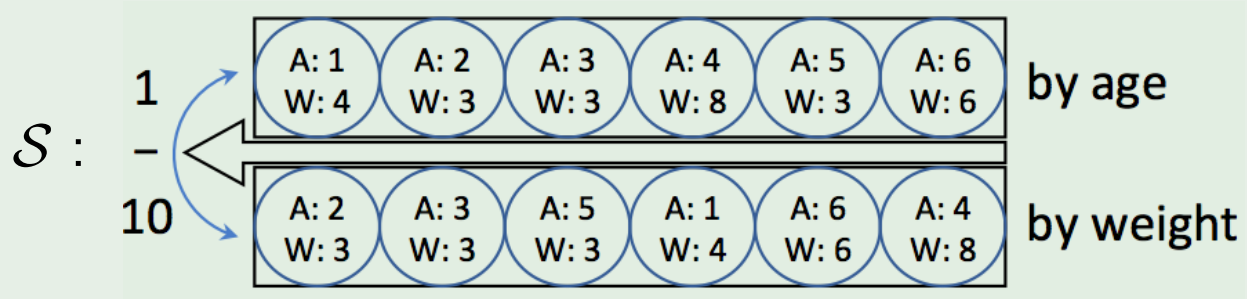
Example (Organizing *Passive* via two priority queues)



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ENIGMA-style

The core idea

Learn to recognize and prefer for selection clauses that look like those that contributed to a proof in past successful runs.

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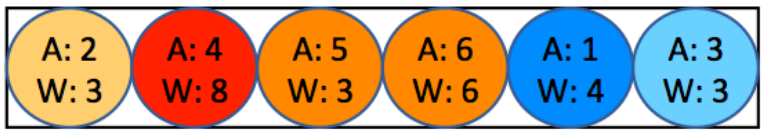
Possible Ways of Integrating the Learnt Advice

Priority:

- sort by model's Y/N and tiebreak by age
- 

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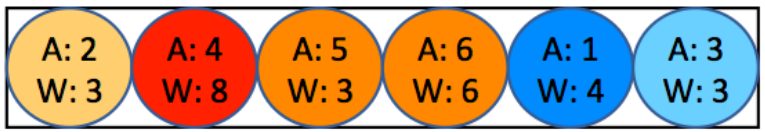
- 

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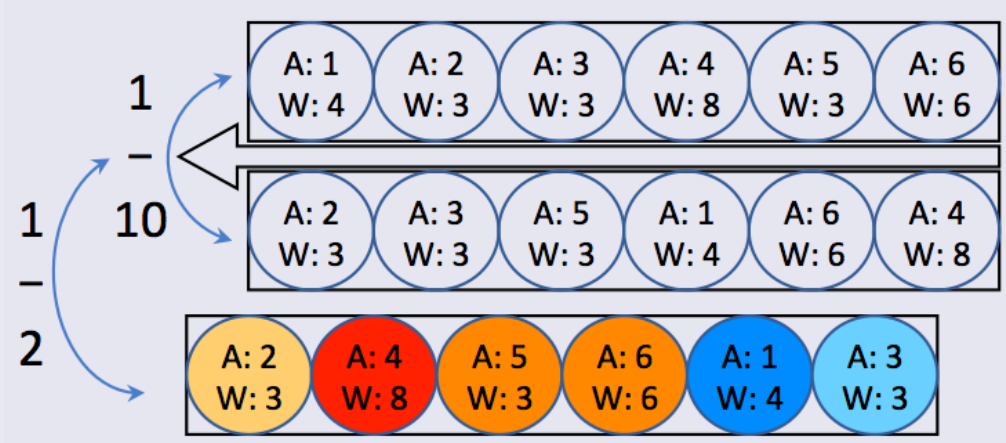
$\mathcal{S} \oplus \mathcal{M}^{1,0}$:



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➥ TRAIL [Crouse et al.'21], [McKeown'23], [Shminke'23], ...

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- for $q_\pi(s,C)$ we simply pick $\mathbb{I}_{\text{Did } C \text{ show up in the found proof?}}$

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Starts with:

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Integrating the learned advice:

- ENIGMA-style: combine with your original heuristic
- RL-inspired: One queue to rule them all!

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Model:

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Iterative improvement:

- Both: yes (ENIGMA calls it "looping")

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