INTRODUCTION TO EYE TRACKING FOR MATH

PART I (SET-UP, ...)

PART II (ANALYSIS, ...)

Hochschule Neu-Ulm
University of Applied Sciences

NEU-ULM
UNIVERSITY OF APPLIED SCIENCES

ANDREA KOHLHASE,
PROFESSOR OF HUMAN-COMPUTER INTERACTION AND WEB DEVELOPMENT

CONTENT OVERVIEW

INTRODUCTION TO EYETRACKING FOR MATH → PART II

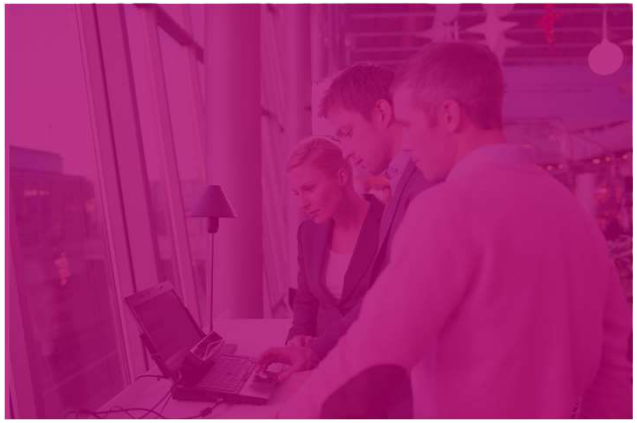
1. SHORT RECALL OF PART I
2. RESEARCH RESULTS FOR MATH
3. CONDUCT AN EYE TRACKING EXPERIMENT
4. IMPLICATIONS FOR MATH

EYE TRACKING

Reminder: What is it?

METHODS: EYE TRACKING
Eye-mind hypothesis

- An observation method to learn
  - Where a person is looking (at any given time)
  - In which order a person is looking at a given stimulus
  - How long a person is looking at one spot



- Eye trackers can be used
  - Stationary in a lab
  - Mobile laptop setting
  - Virtual Reality
    (even as input modality)
  - With glasses: „everywhere“

PLAN YOUR STUDY: EYE TRACKING

Metrics provided by an eye tracker

- Where a person is looking (at any given time)
  → Point-of-Gaze (Location)
- In which order a person is looking at a given stimulus
  → Order of fixations
- How long or how often a person is looking at one spot
  → Fixation counts
  → Visit counts
  → Duration of fixation in an area-of-interest
- When and how often a person blinks → Blink rate
- How the eyes react with respect to stimulus conditions
  → Pupil dilation, saccadic intrusions

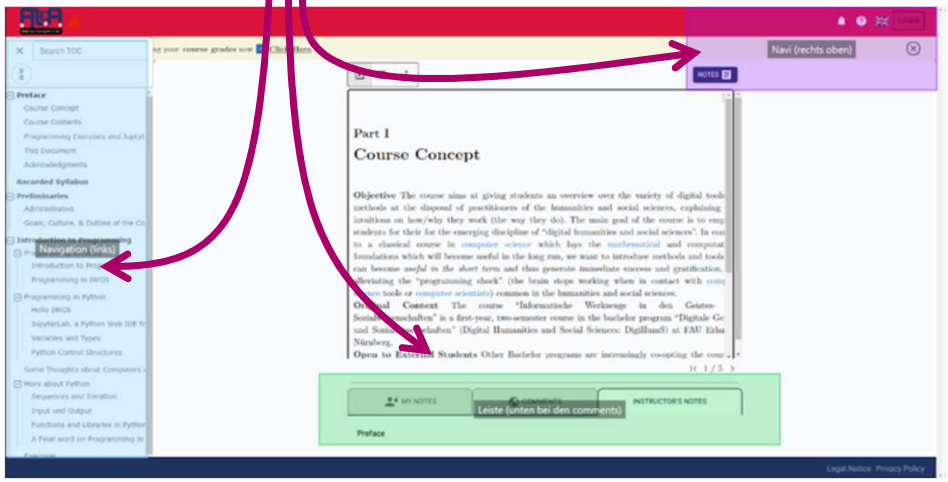
analyses are possible for any set among the participants

see [Poole&Ball: Eye Tracking in HCI and Usability Research: Current Status and Future Prospects]

EYE TRACKING DATA

For statistical analysis

Self-defined
Areas of Interest (AoI)

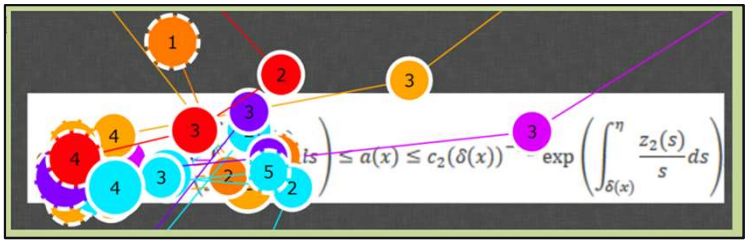


	Navi (rechts oben)
Information	
AOI duration (ms)	72530.2
AOI duration (%)	100
Size (cm2)	60.1
Size (%)	4.1
Respondent base	17
Fixation based metrics	
Respondent ratio (%)	94.1
Revisit count	2.8
Fixation count	49.2
TTFF AOI (ms)	16496.7
Dwell time (ms)	1643
Dwell time (%)	1.9
First fixation duration (ms)	40.6
Saccade based metrics	
Respondent ratio (%)	94.1
Saccade count	46
Amplitude (deg)	1.2
Peak velocity (deg/s)	73.9

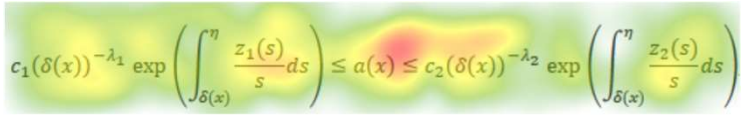
VISUAL ANALYSIS TOOLS

Based on the data from an eye tracking experiment

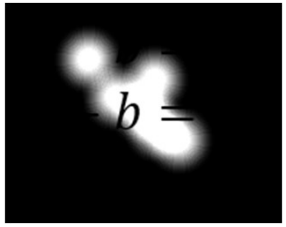
- Eye tracking technology → close observation
  - Gaze plots



  - Heat maps



  - Spotlight maps



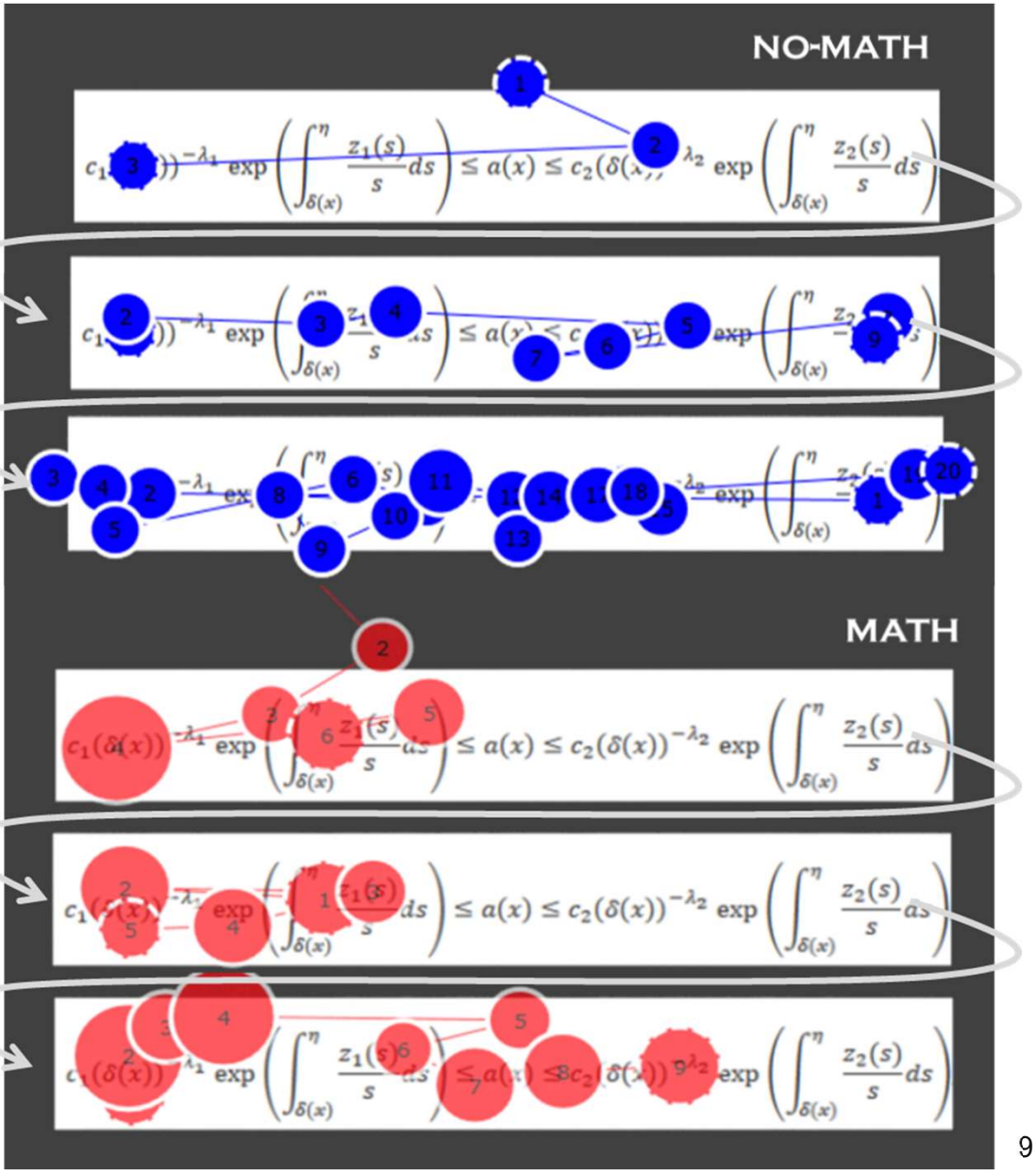
  - Statistical data analysis: fixation count, visit count, …

TEXT OR IMAGE?

What we found out …

- math-oriented (MATH)
- non-math-oriented people (NO-MATH)

A participant's gazeplot



Visual **Structure** ⟵

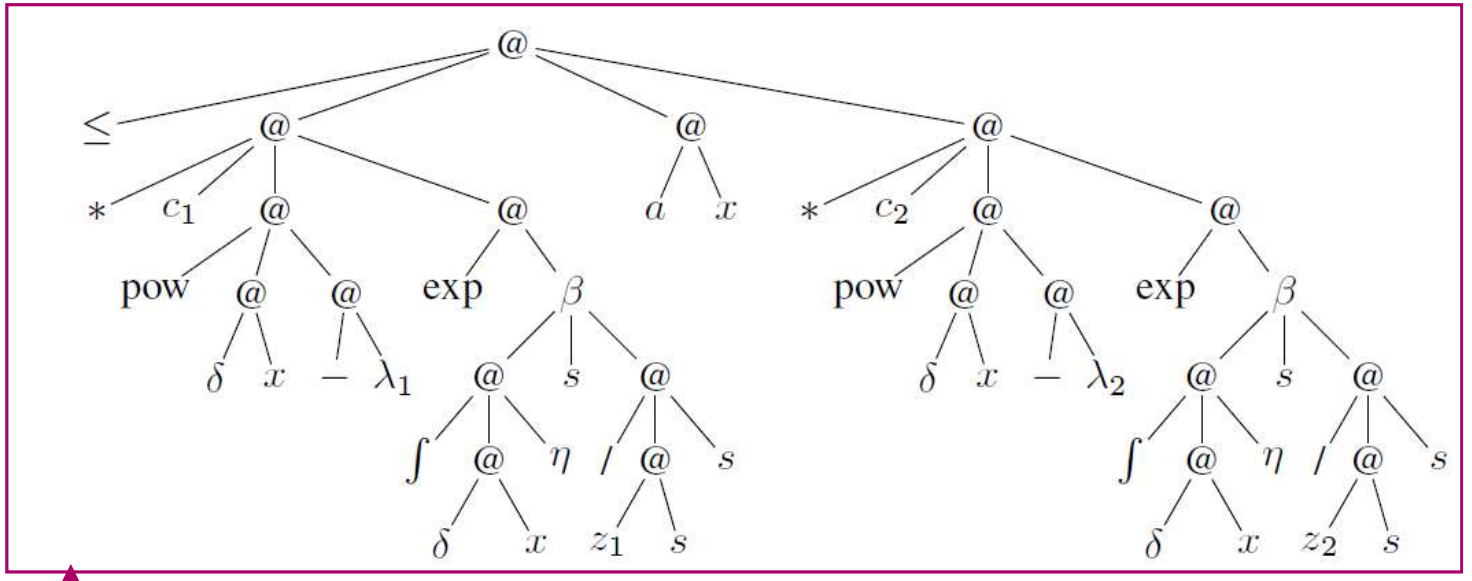
Math Literacy! ⟵

→ MATH LITERACY!

What is the general decoding approach?

DECODING OF MATH EXPRESSIONS

A depth-first traversal of the operator tree



$$c_1(\delta(x))^{-\lambda_1} \exp\left(\int_{\delta(x)}^{\eta} \frac{z_1(s)}{s} ds\right) \leq a(x) \leq c_2(\delta(x))^{-\lambda_2} \exp\left(\int_{\delta(x)}^{\eta} \frac{z_2(s)}{s} ds\right)$$

Andrea Kohlhase, Michael Kohlhase, and Michael Fürsich: *Visual Structure in Mathematical Expressions*, 2017

DECODING OF MATH EXPRESSIONS

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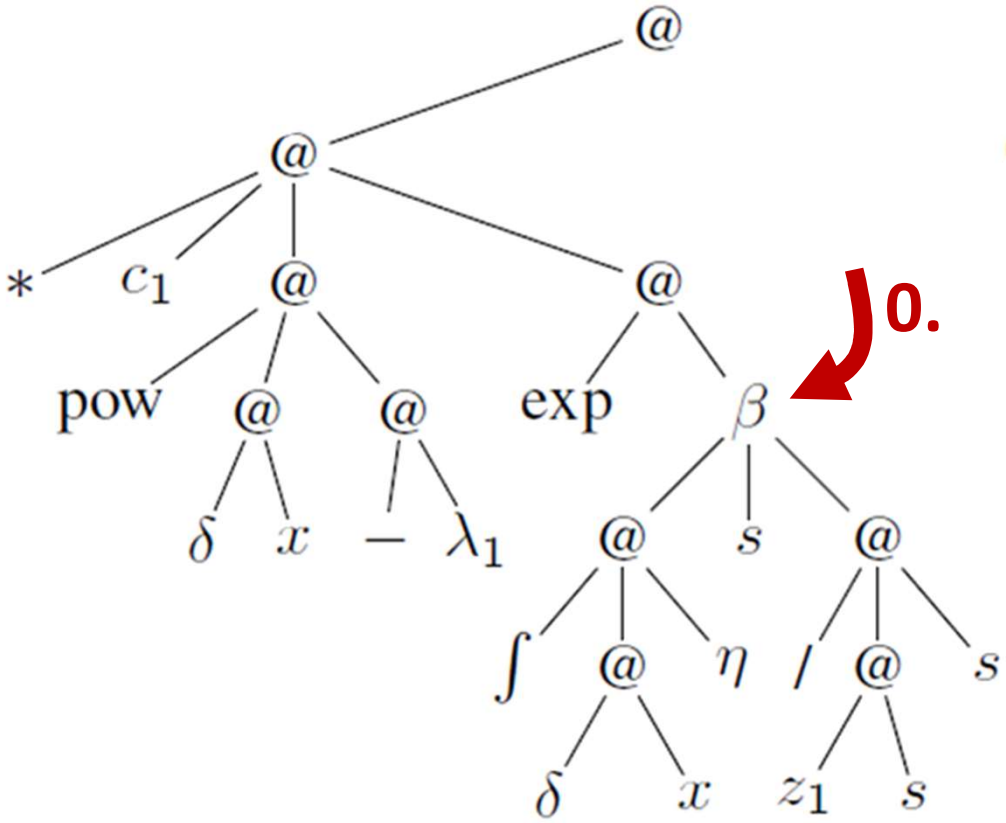

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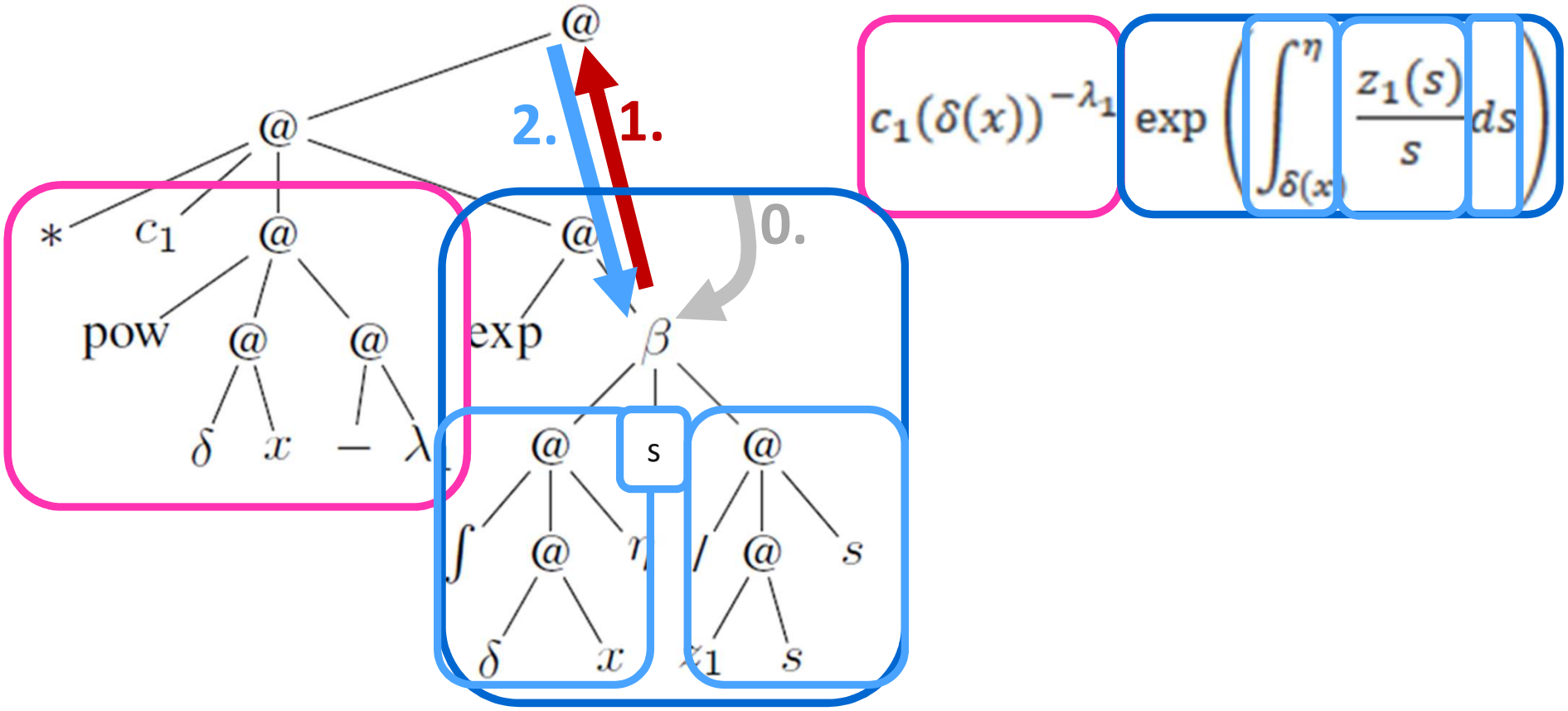

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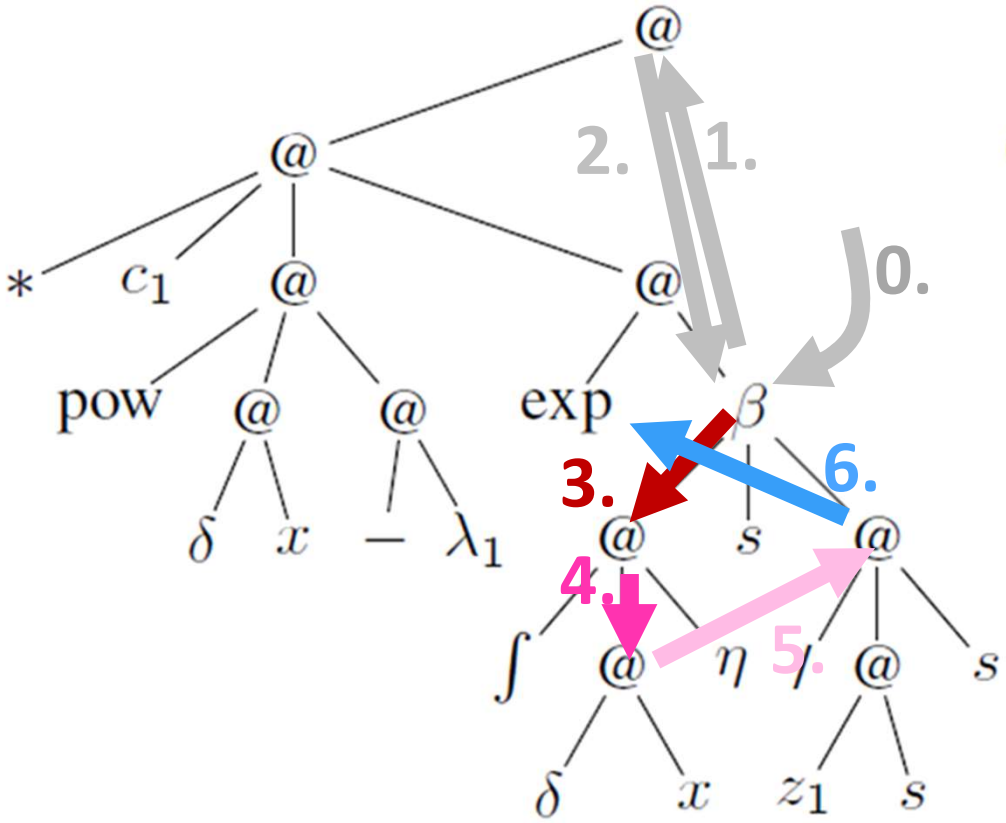

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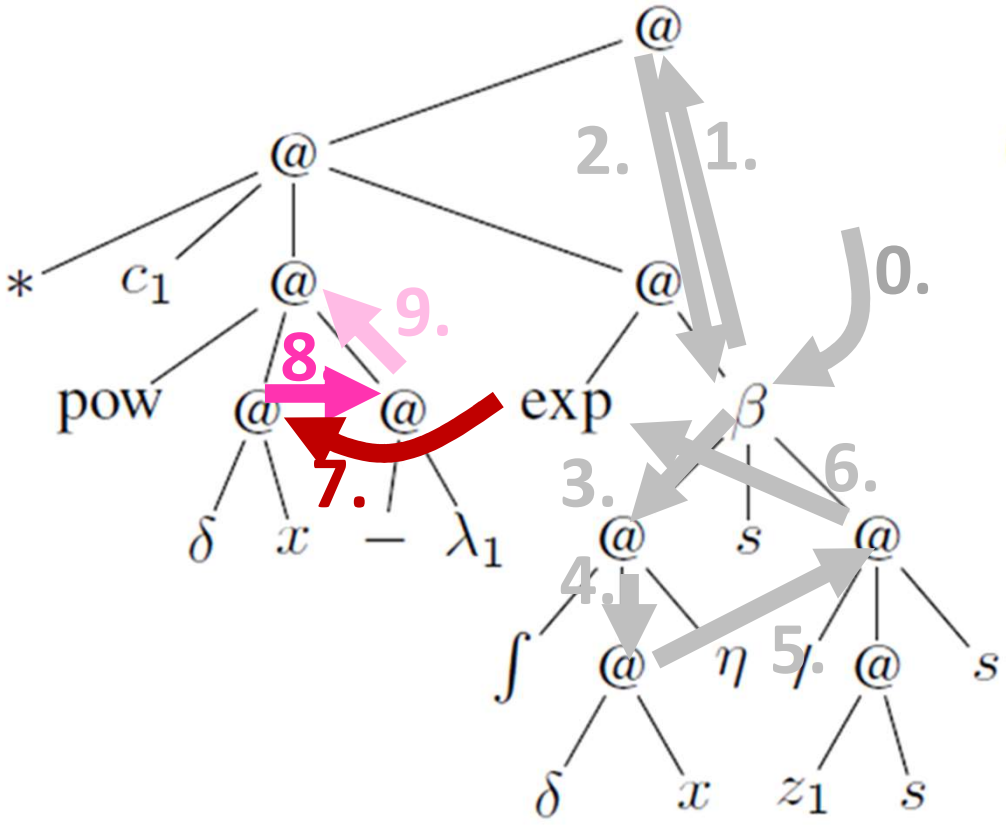

$$c_1(\delta(x))^{-\lambda_1} \exp\left(\int_{\delta(x)}^{\eta} \frac{z_1(s)}{s} ds\right)$$

→ The compositional approach of decoding is a math practice!

Andrea Kohlhase, Michael Kohlhase, and Michael Fürsich: *Visual Structure in Mathematical Expressions*, 2017

SCOPE OF MATH LITERACY?

Is math literacy limited to math expressions?

DECODING OF MATH/TECH DOCUMENTS

Participants were asked to read a text and write a summary

1 Radioactive Decay

Background In September 1991 the famous Iceman (Ötzi), a mummy from the Neolithic period of the Stone Age found in the ice of the Ötztal Alps (hence the name Ötzi) in Southern Tyrolia near the Austrian-Italian border, caused a scientific sensation.

Problem When did Ötzi approximately live and die if the ratio of carbon ${}_6C^{14}$ to carbon ${}_6C^{12}$ in this mummy is 52.5%?

Physical Information In the atmosphere and in living organisms, the ratio of radioactive ${}_6C^{14}$ (made radioactive by cosmic rays) to ordinary ${}_6C^{12}$ is constant. When an organism dies, its absorption of ${}_6C^{14}$ by breathing and eating terminates. Hence one can estimate the age of a fossil by comparing the radioactive carbon ratio in the fossil with that of the atmosphere. To do this one needs to know the half-life of ${}_6C^{14}$, which is 5715 years.

Solution Radioactive decay is governed by the ODE $y' = ky$. By separation and integration (where t is time and y_0 is the initial ratio of ${}_6C^{14}$ to ${}_6C^{12}$)

$$\frac{dy}{y} = k\,dt, \qquad \ln|y| = kt + c, \qquad y = y_0 e^{kt}$$

Next we use the half-life $H = 5715$ to determine k. When $t = H$, half of the original substance is still present, thus

$$kH = \ln 0.5 \qquad 0.693$$

Biomechanical/Electrical Engineering Program at SWU, Thailand

Understanding Levels:

- LOW
- HIGH

Taweechai Ouypornkochagorn: *Discourse Phenomena in Math Documents*, 2018

DECODING OF MATH/TECH DOCUMENTS

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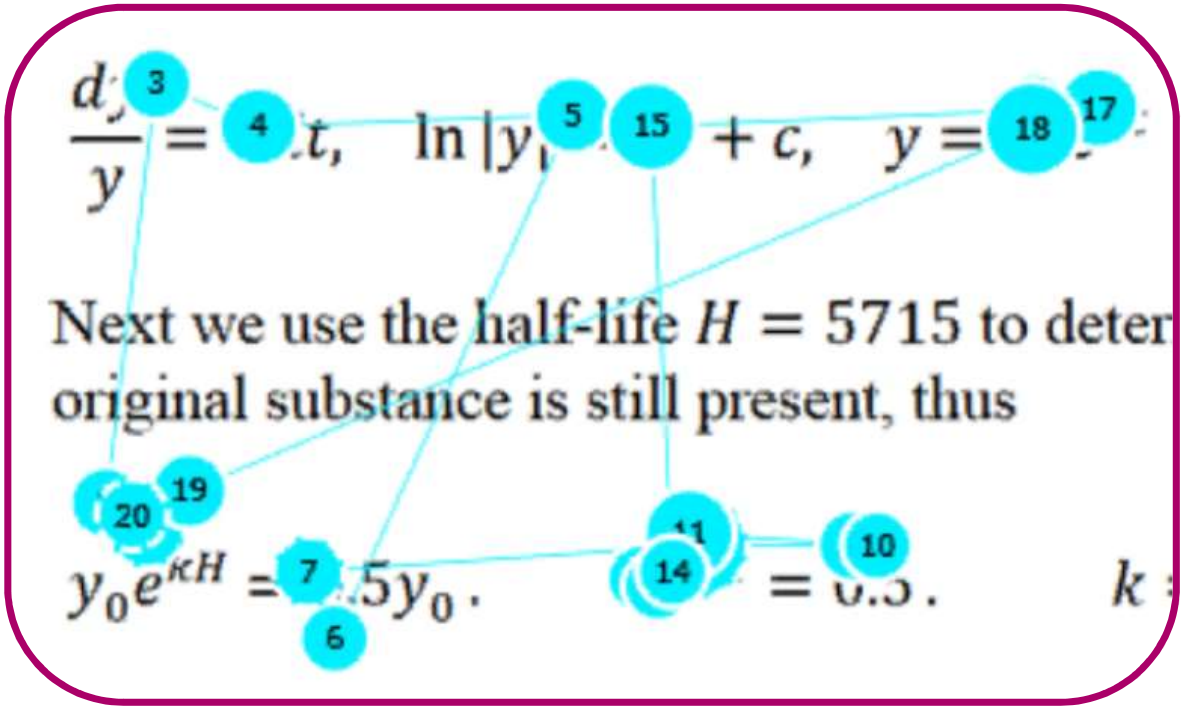


→ Reading flow:

1. Local regressions wrt. to equation in focus
   - i. Identifier declarations
   - ii. Deriving equations
   - iii. Justifications
2. Non-local regressions

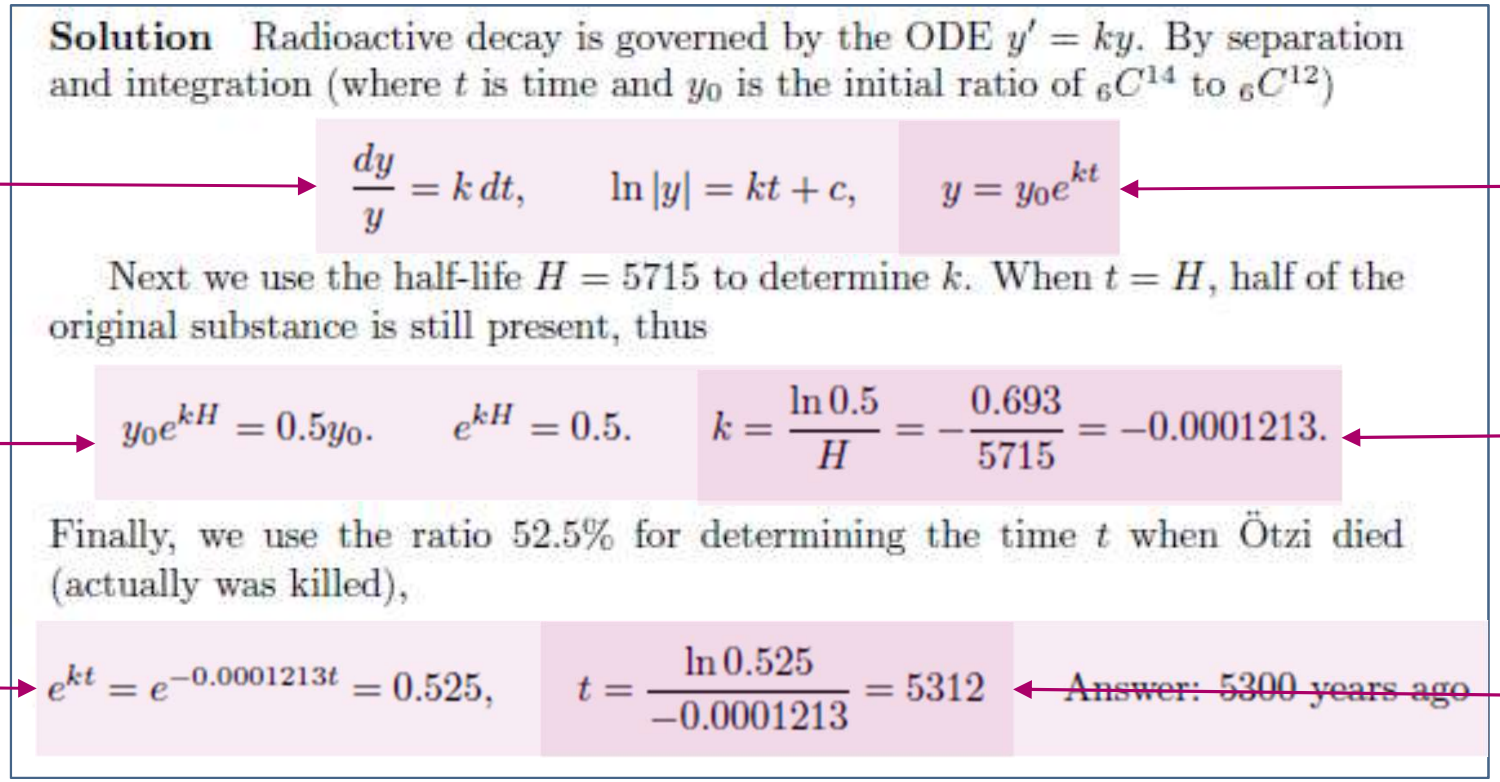
Andrea Kohlhase, Michael Kohlhase, and Taweechai Ouypornkochagorn: *Discourse Phenomena in Math Documents*, 2018

MATH COMPETENCY?

Does math literacy vary with math competency?

DECODING OF MATH/TECH DOCUMENTS

Summary quality → understanding level → math competency



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Next we use the half-life $H = 5715$ to determine k. When $t = H$, half of the original substance is still present, thus

$$y_0 e^{kH} = 0.5 y_0. \qquad e^{kH} = 0.5. \qquad k = \frac{\ln 0.5}{H} = -\frac{0.693}{5715} = -0.0001213.$$

Finally, we use the ratio 52.5% for determining the time t when Ötzi died (actually was killed),

$$e^{kt} = e^{-0.0001213t} = 0.525, \qquad t = \frac{\ln 0.525}{-0.0001213} = 5312$$

~~Answer: 5300 years ago~~

Andrea Kohlhase, Michael Kohlhase, and Taweechai Ouypornkochagorn: *Discourse Phenomena in Math Documents*, 2018

DECODING OF MATH/TECH DOCUMENTS

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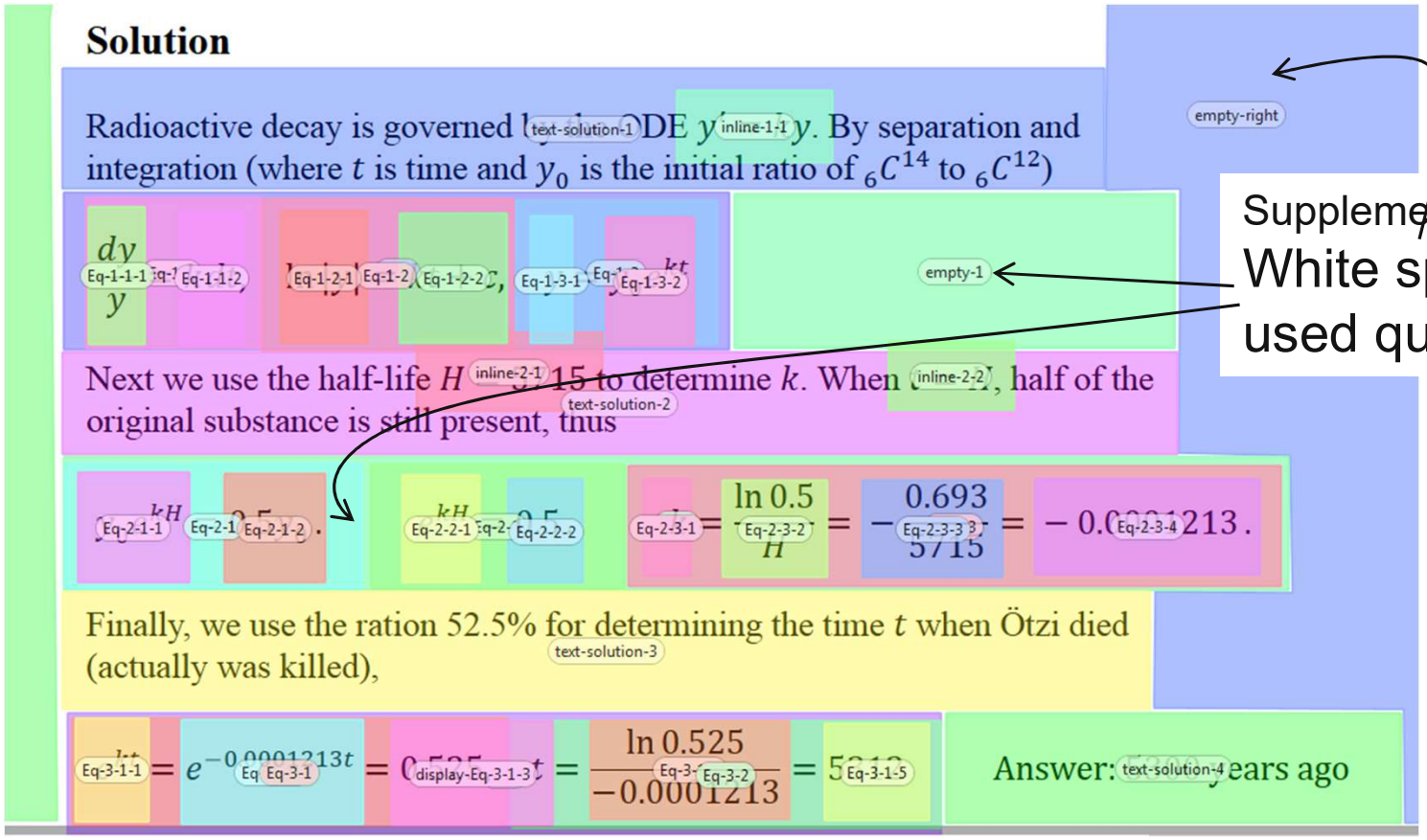


Supplementary Note:
White space was used quite a lot

Andrea Kohlhase, Michael Kohlhase, and Taweechai Ouypornkochagorn: *Discourse Phenomena in Math Documents*, 2018

MATH COMPETENCY

Normalized Visit duration per understanding level

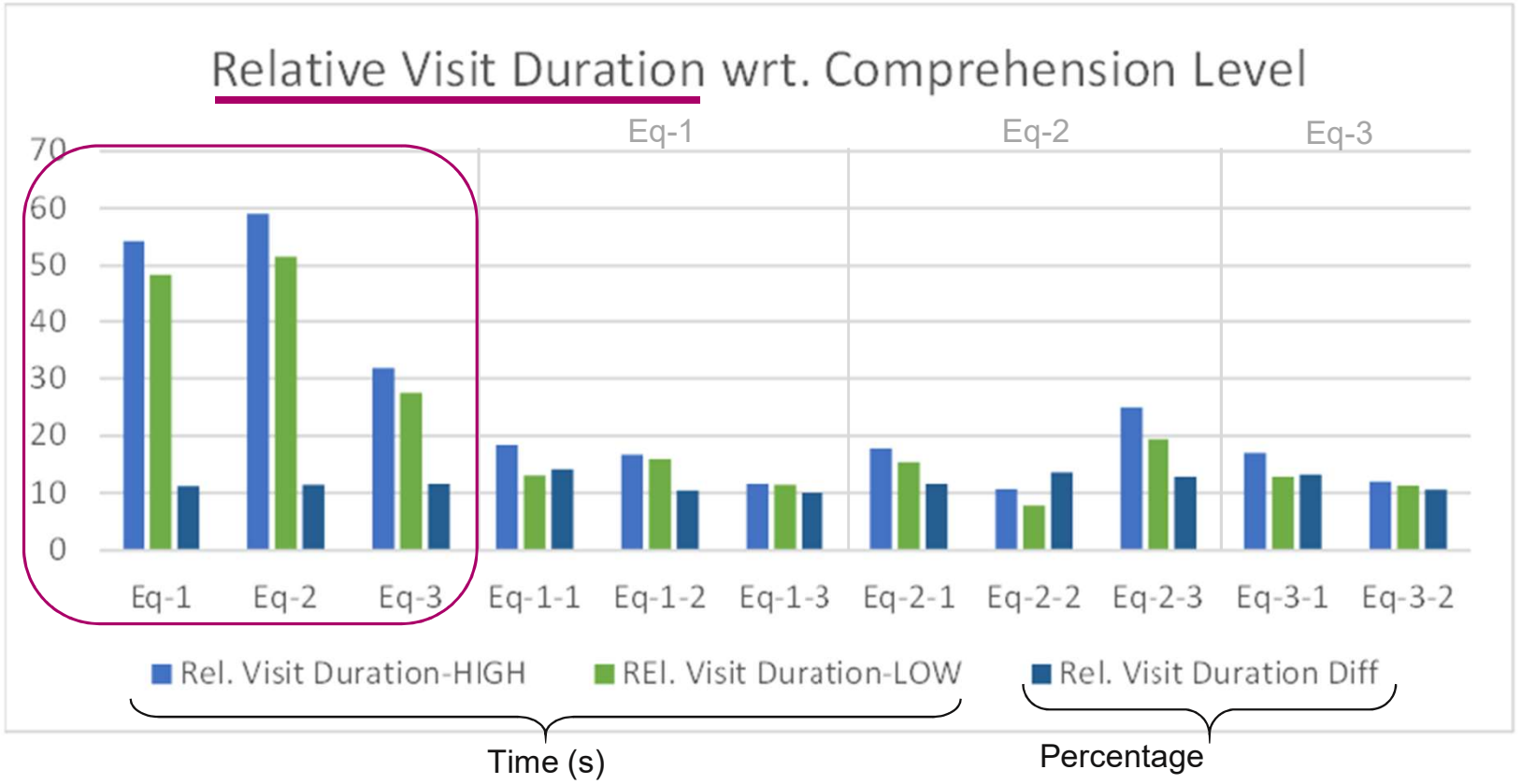
Understanding Levels:

- LOW
- HIGH

just sligthly higher

HIGH ~ LOW: ← Visit duration

→ No real time difference when reading



Andrea Kohlhase, Michael Kohlhase, and Taweechai Ouypornkochagorn: *Discourse Phenomena in Math Documents*, 2018

MATH COMPETENCY

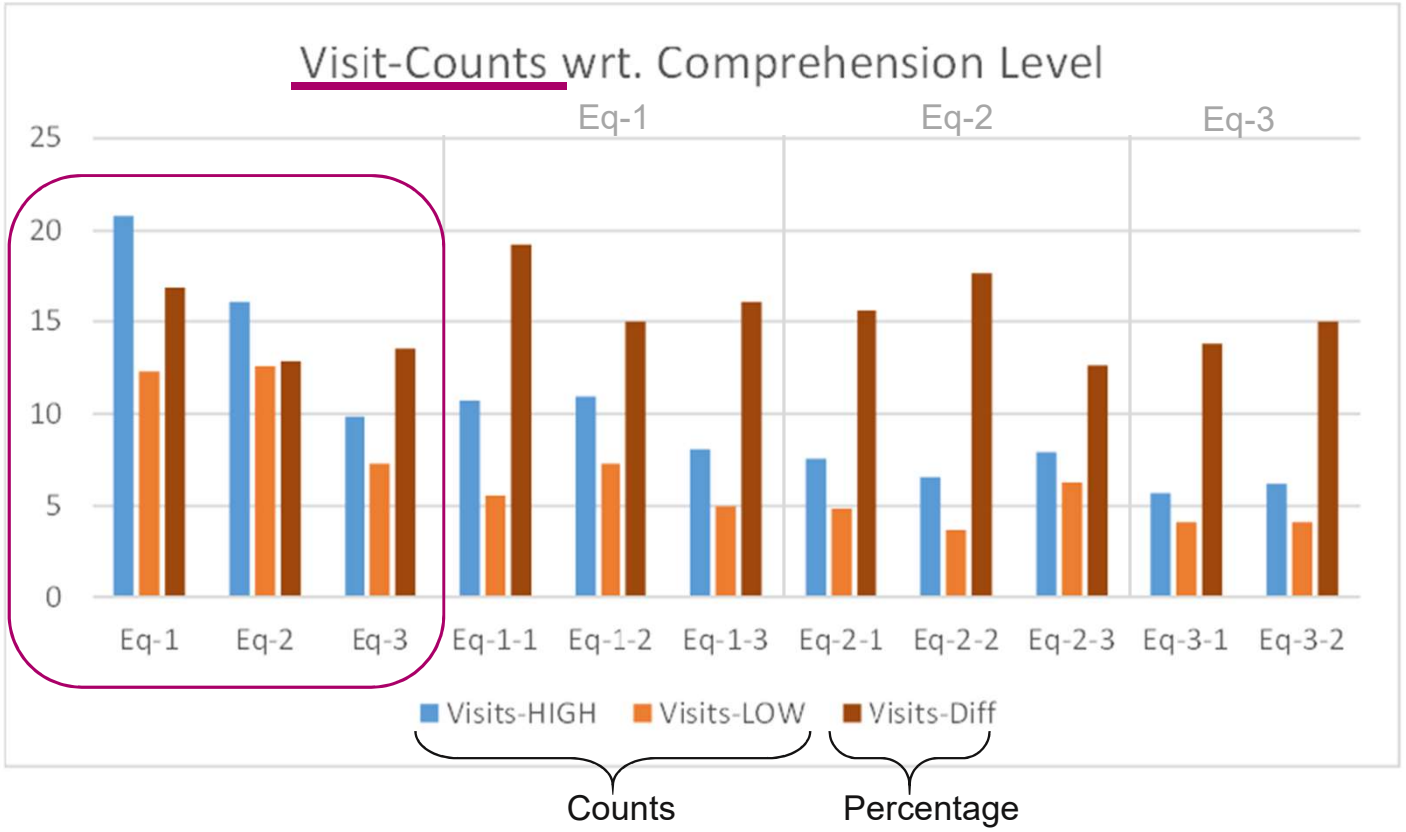
Visits per understanding level

Understanding Levels:

- LOW
- HIGH

HIGH > LOW ⟵

- HIGH is more active in global decoding
- Activity of all decreases within the solution



Andrea Kohlhase, Michael Kohlhase, and Taweechai Ouypornkochagorn: *Discourse Phenomena in Math Documents*, 2018

MATH COMPETENCY

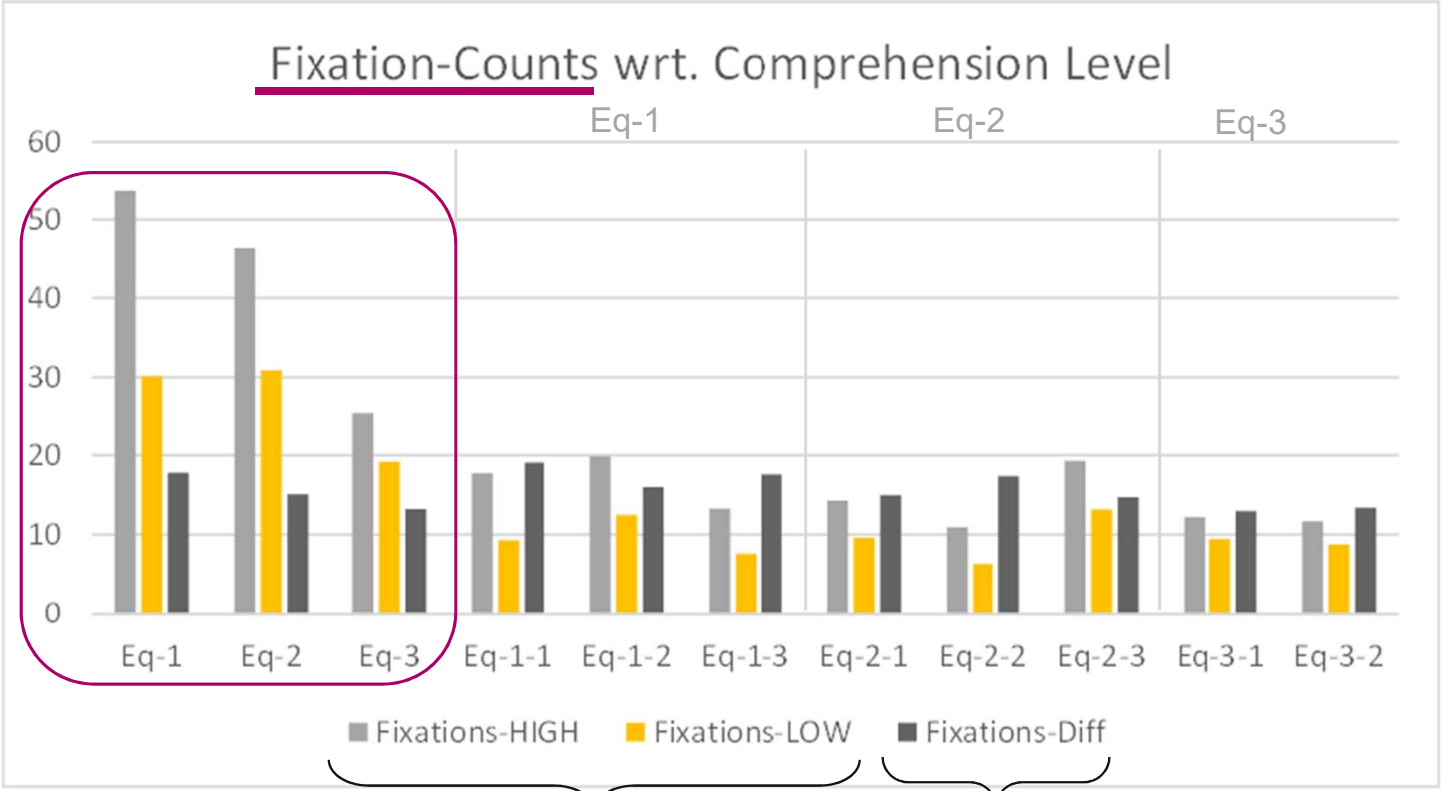
Fixations per understanding level

Understanding Levels:

- LOW
- HIGH

HIGH >> LOW ←

- HIGH is really more active in local decoding
- Activity of all decreases within the solution



→ Math competency correlates with the activity level of visual scanning, but not time!

HOME OF MATH EXPRESSIONS?

Is math literacy different
for different disciplines?

COMMUNITIES OF PRACTICE (COP)

Looking for visual patterns (while discovering errors)

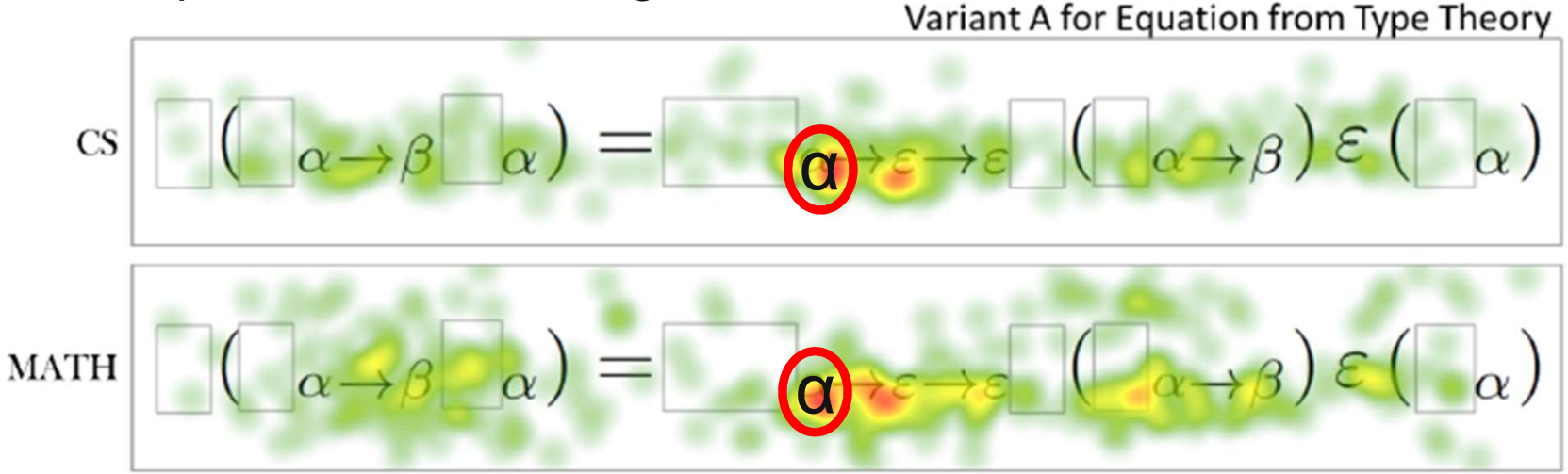
= organized groups of people with a common concern/interest in a specific domain

Original Equation from Type Theory

$$\varepsilon\left(F_{\alpha\to\beta}A_{\alpha}\right)=app_{\varepsilon\to\varepsilon\to\varepsilon}\varepsilon\left(F_{\alpha\to\beta}\right)\varepsilon\left(A_{\alpha}\right)$$

Is this a faithful representation of the given formula?

Variant A for Equation from Type Theory



→ ?

Andrea Kohlhase: *Factors for Reading Mathematical Expressions*, 2016

COMMUNITIES OF PRACTICE (COP)

COP membership and error validation: *Computational Math*

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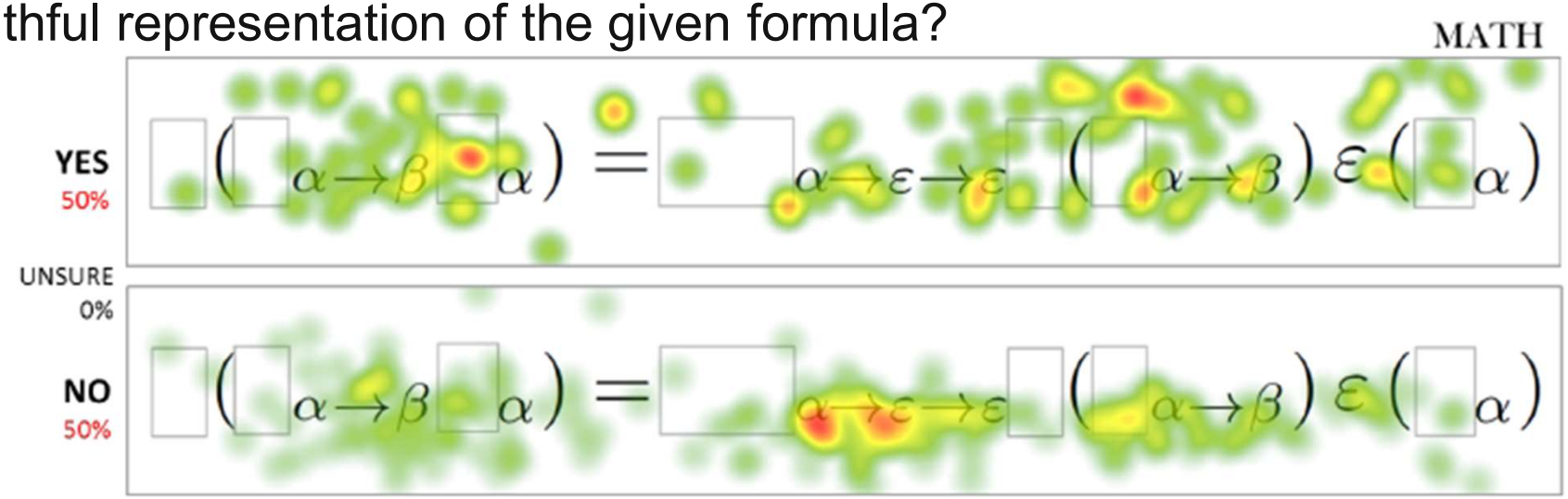

Andrea Kohlhase: *Factors for Reading Mathematical Expressions*, 2016

COMMUNITIES OF PRACTICE (COP)

COP membership and error validation: *Type Theory (CS)*

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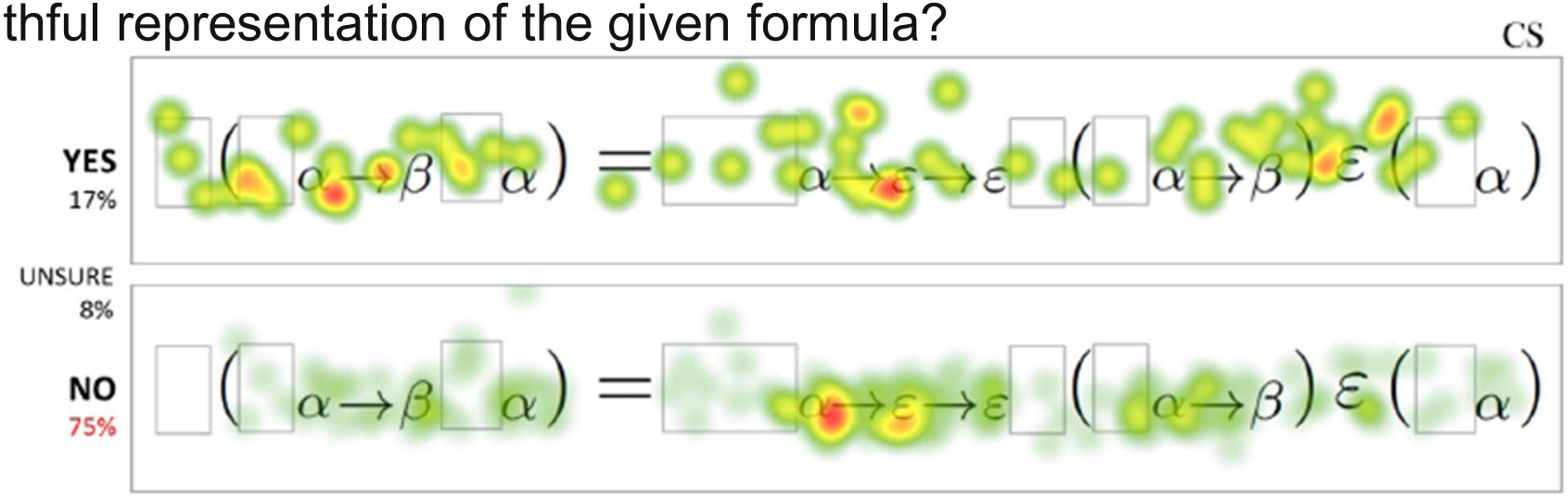


Andrea Kohlhase: *Factors for Reading Mathematical Expressions*, 2016

COMMUNITIES OF PRACTICE (COP)

Result: competency/COP membership

- **Competency** correlates with efficient visual scanning (for errors)
- **COP membership** correlates with the visual scanning process

The visual scanning process might indicate competency or COP membership

→ **Can we deduce (COP) competency from the visual scanning process or activity level?**

→ **E.g., build a learner model from eye gazing behavior?**

Andrea Kohlhase: *Factors for Reading Mathematical Expressions*, 2016

LET‘S DO AN EXPERIMENT …

Now!

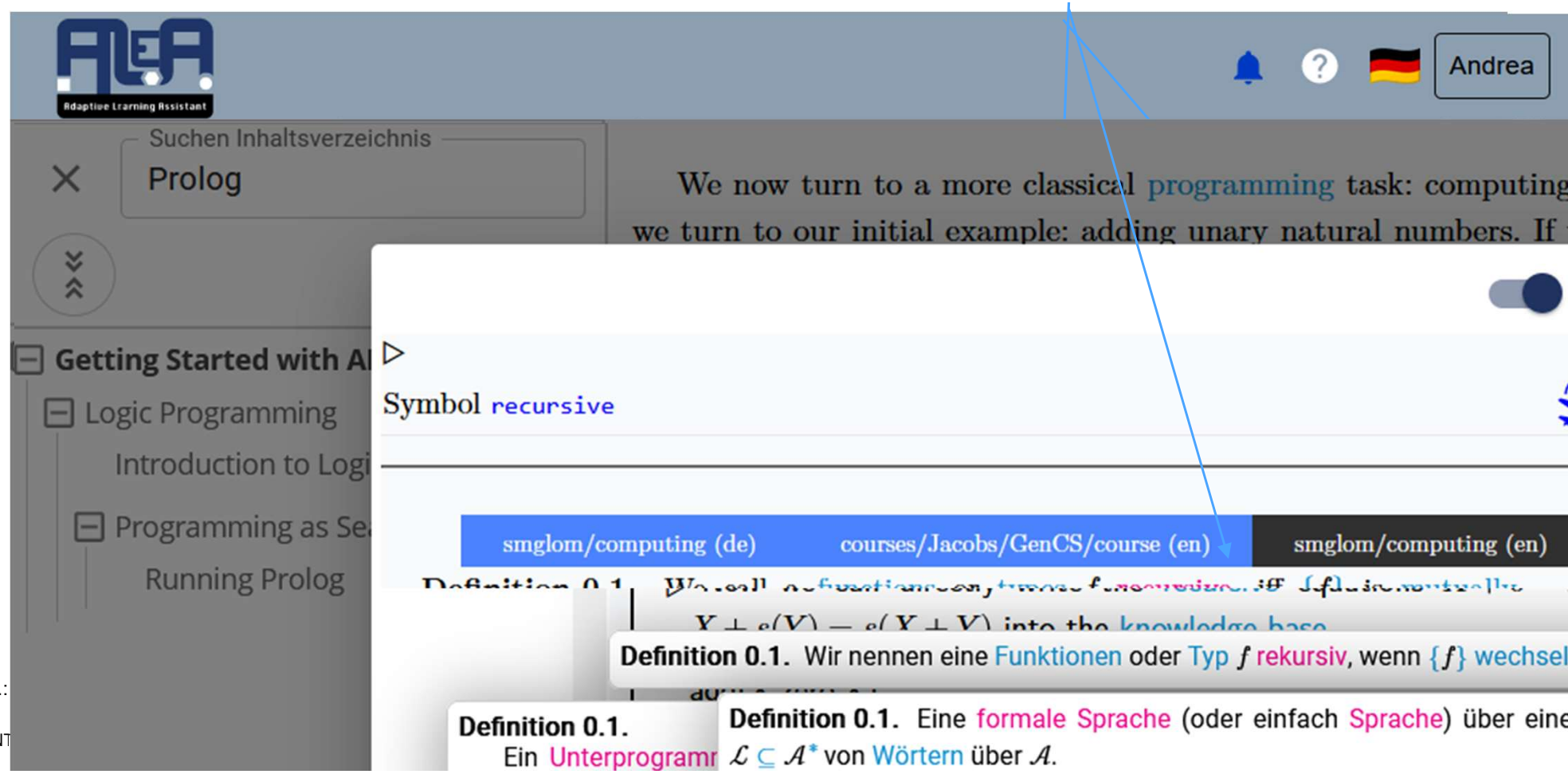
TOWARDS FACILITATING LEARNING

Formalization of Practices
→ Learning Opportunities

ALEA

Adaptive Learning Assistant

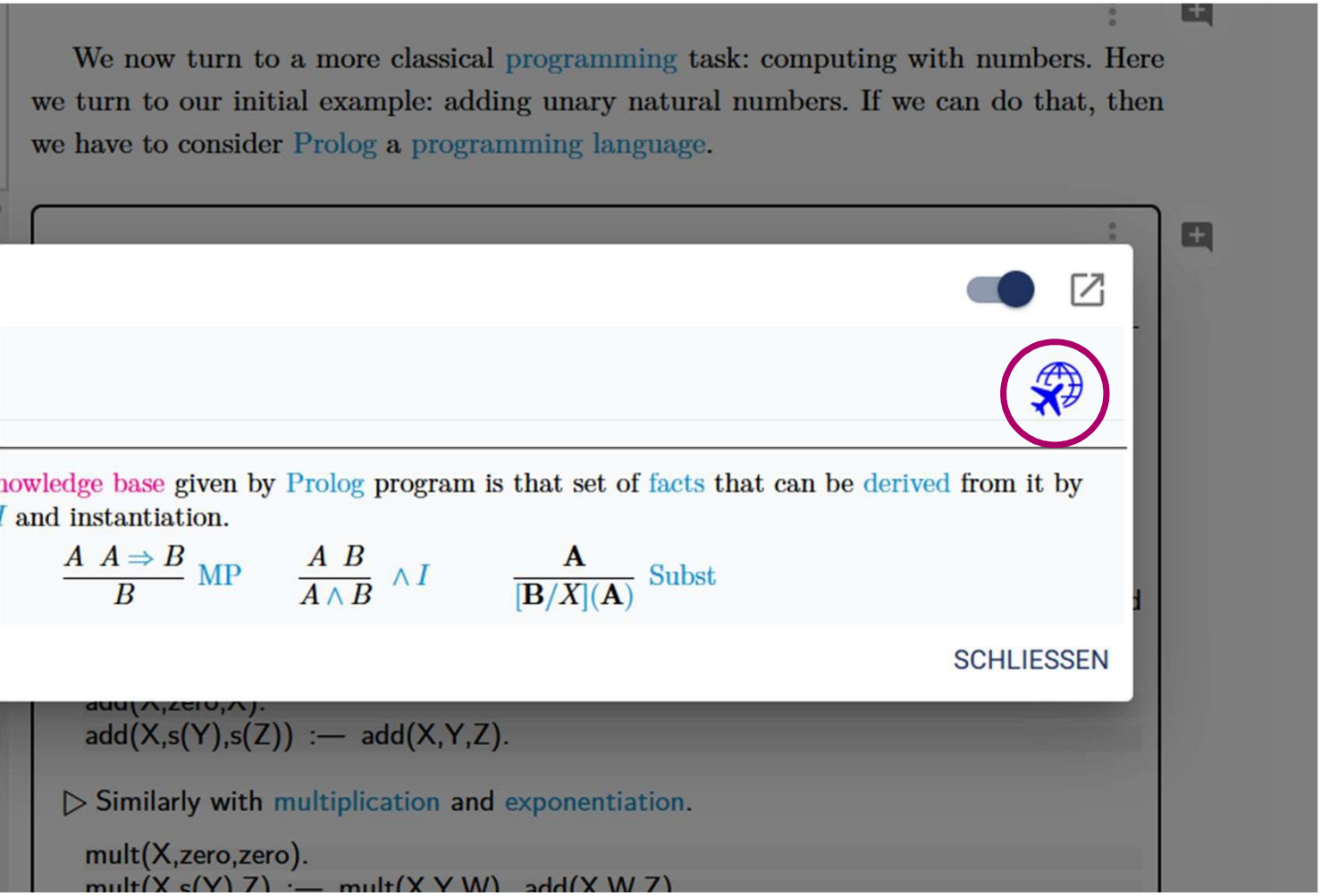
https://courses.voll-ki.fau.de

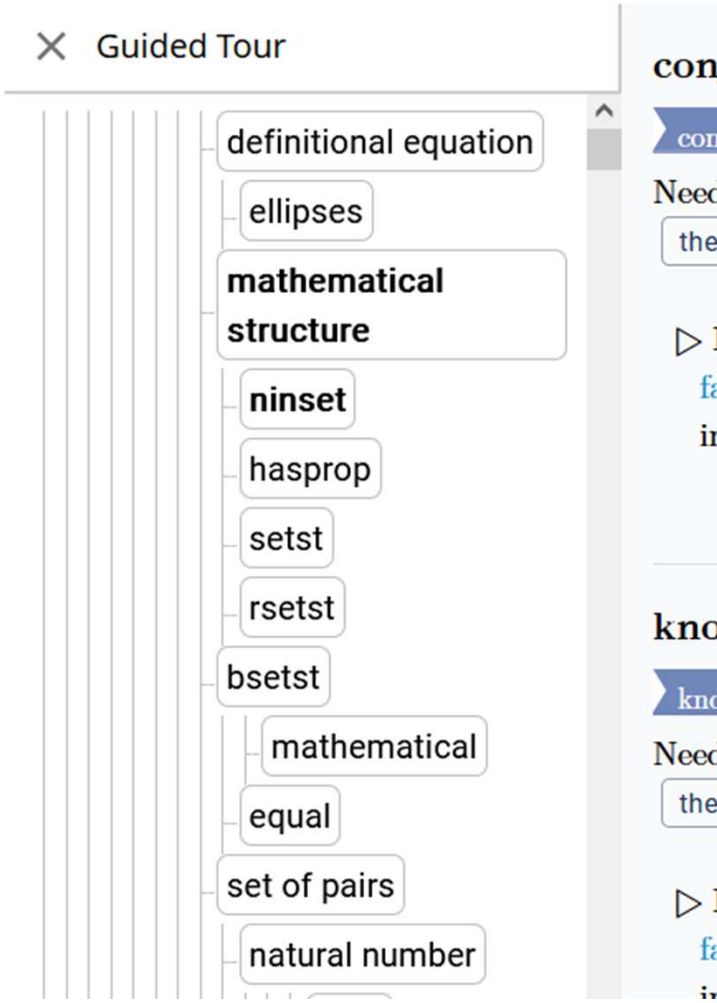


ALEA

Adaptive Learning Assistant: Guided Tour

https://courses.voll-ki.fau.de



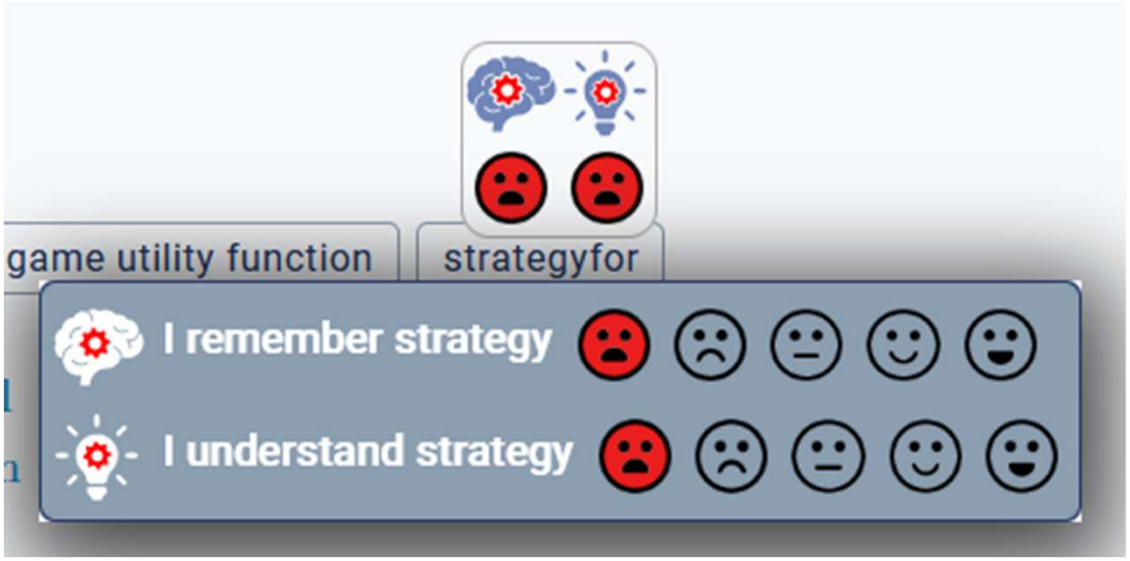


ALEA

Adaptive Learning Assistant**: feeding on learner model**

For each learner a learner model is maintained,
that stores information about activity in the learning material
and self-assessed understanding levels (based on Bloom‘s taxonomy for learning objectives)

ALeA is a great playground for automatic deduction of understanding levels via eyetracking



Marc Berges et al.: *Learning Support Systems based on Mathematical Knowledge Managment*, 2023

FORMALIZATION OF MATH KNOWLEDGE

Decoding of math expressions can be generalized to text

Flexi-formalization

- A formal encoding in a tech document

Let X be a set, then a set system $\mathcal{O}$ is called a **topology**, iff

1. $\emptyset \in \mathcal{O}$
2. $X \in \mathcal{O}$
3. If $S \subseteq \mathcal{O}$, then $\bigcup_{s \in S} s \in \mathcal{O}$. (**closed under unions**)
4. If $S \subseteq \mathcal{O}$ is finite, then $\bigcap_{s \in S} s \in \mathcal{O}$. (**closed under finite intersections**)

sTeX

```
\begin{sdefinition}
  \vardef{bset}[bind]{X}\vardef{top}{\mathcal{O}}
  Let $\bset$ be a \sn{set}, then a \sn{set system} $\top$ is called a
  \definame{topology}, iff
  \begin{enumerate}
  \item \inlineass[name=eset.cond]{$\inset\eset\top$}.
  \item \inlineass[name=fullset.cond]{$\inset\bset\top$}.
  \item \inlineass[name=union.con]{$\inset{\munionCollection{s}Ss}\top$
      if $\sseteq{S}\top$}.
  \item \inlineass[name=fincap.cond]{$\inset{\mintersectCollection{s}Ss}\top$
      if $\sseteq{S}\top$ is \sn{finite-cardinality?finite}}.
  \end{enumerate}
\end{sdefinition}
```

Michael Kohlhase and Dennis Müller: *sTeX3 -- A LaTeX-based Ecosystem for Semantic/Active Mathematical Documents*, 2023

FORMALIZATION OF MATH KNOWLEDGE

What can we do with such formalizations?

Let X be a set, then a set system $\mathcal{O}$ is called a **topology**, iff

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The term „topology“ depends on other terms, e.g., set, $\emptyset$, $\bigcup_{s \in S}$, finite $\bigcap_{s \in S}$, …

and those in turn depend on others, etc.

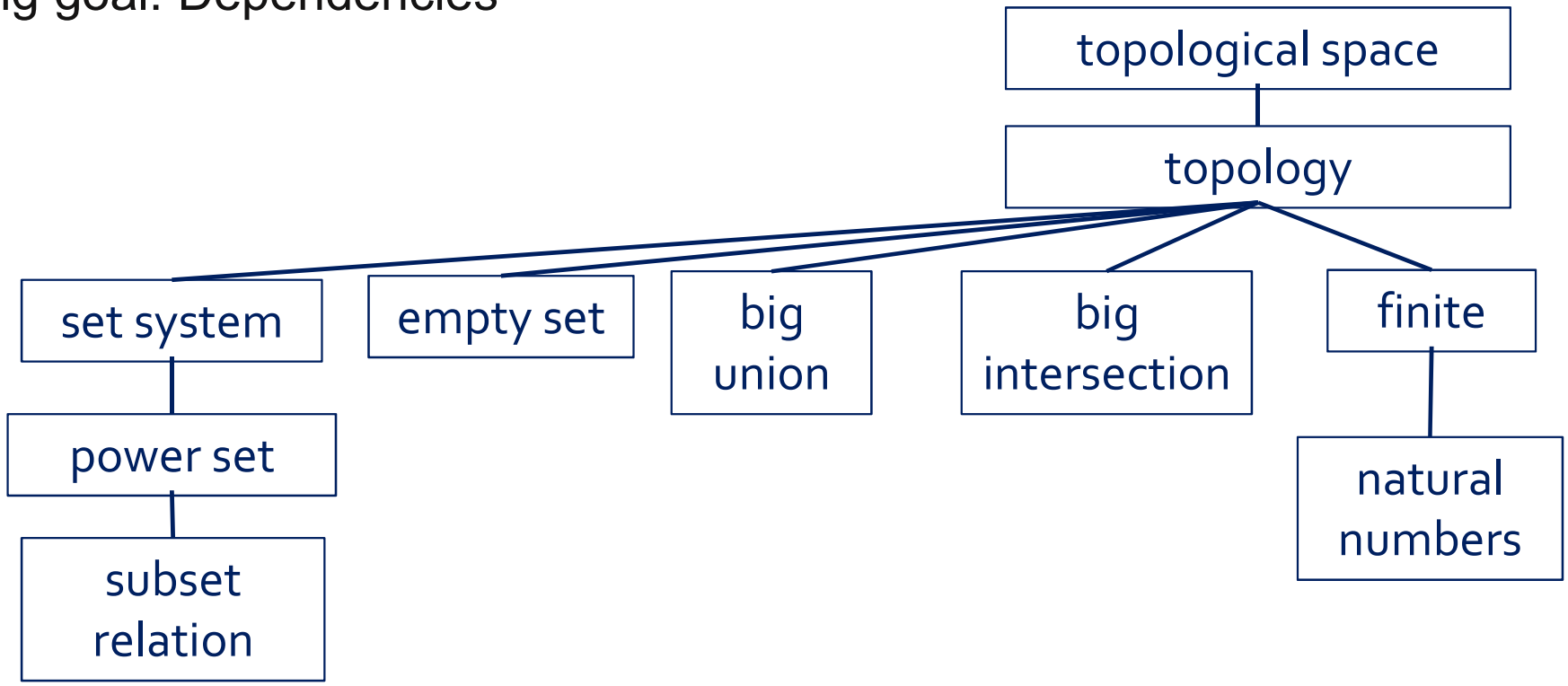
→ We have a **terminological dependency order**

Andrea Kohlhase und Michael Kohlhase: *More Interactions in ALeA –Towards New Added-Value Services based on Semantic Markup*, 2023

FORMALIZATION OF MATH KNOWLEDGE

Terminological dependency order for „topology“

Learning goal: Dependencies



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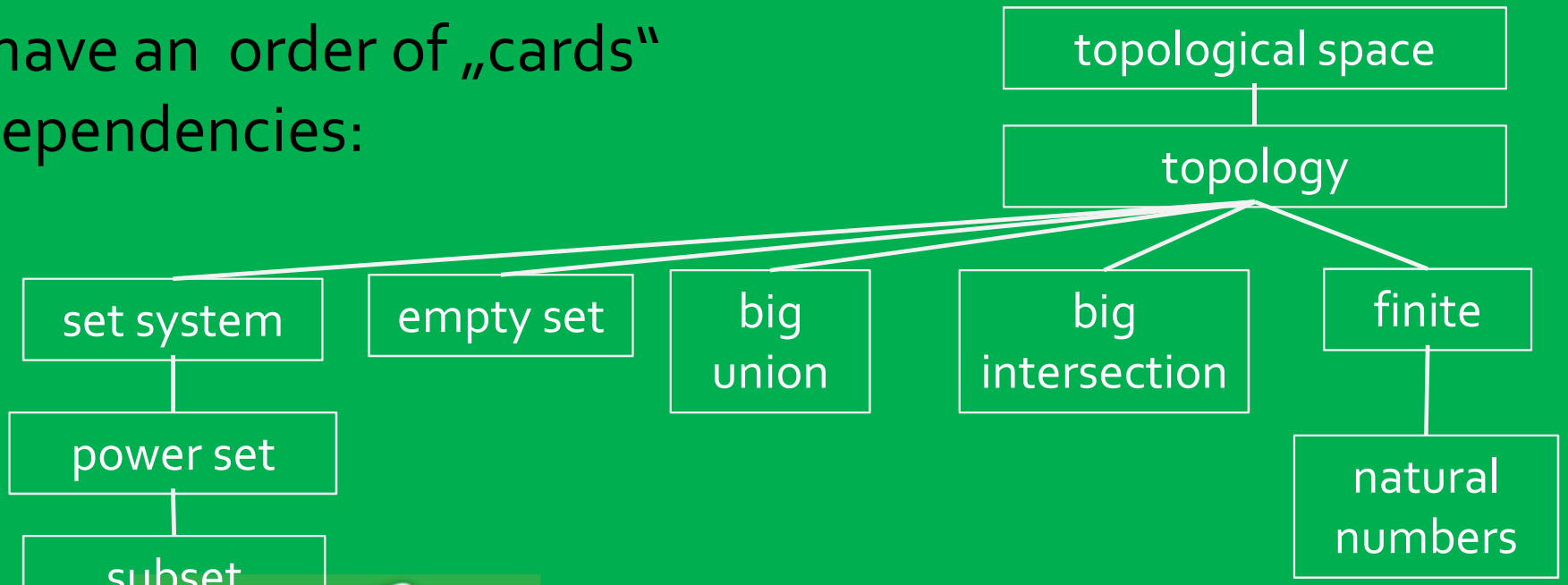
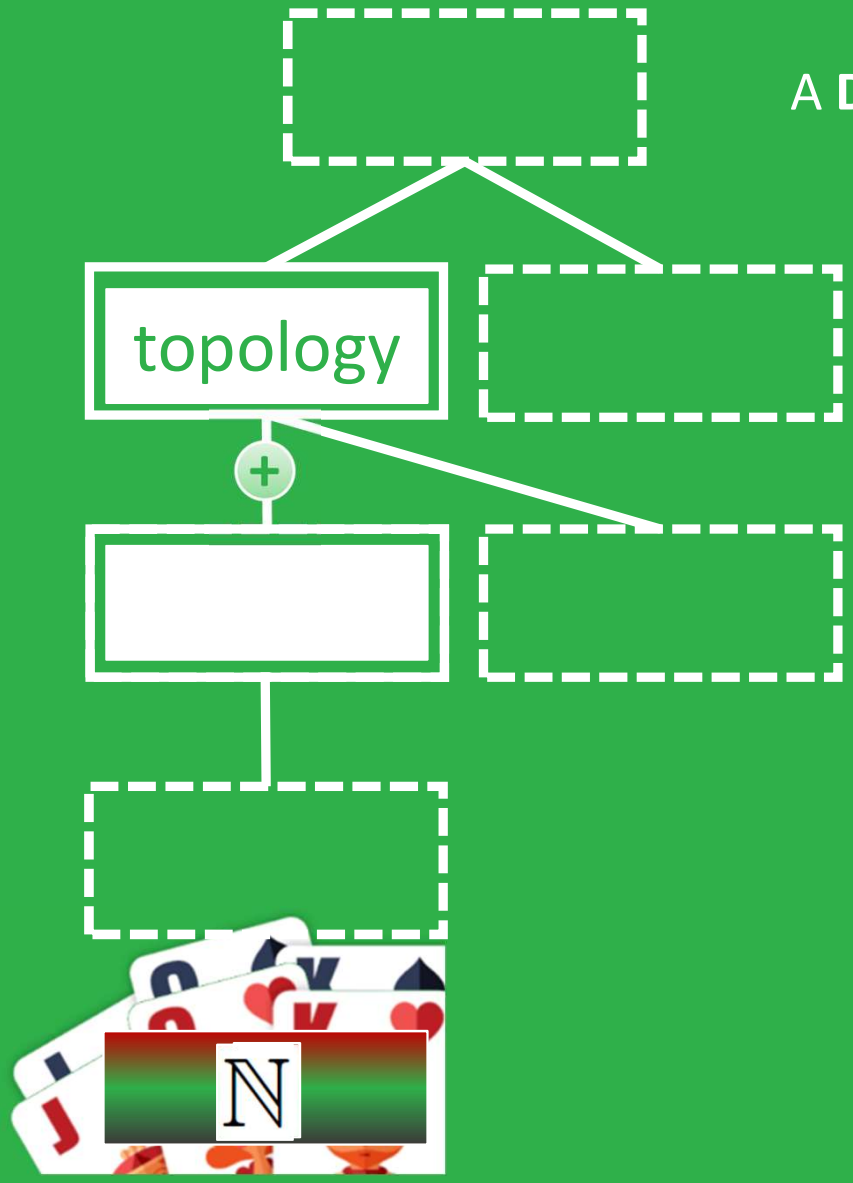
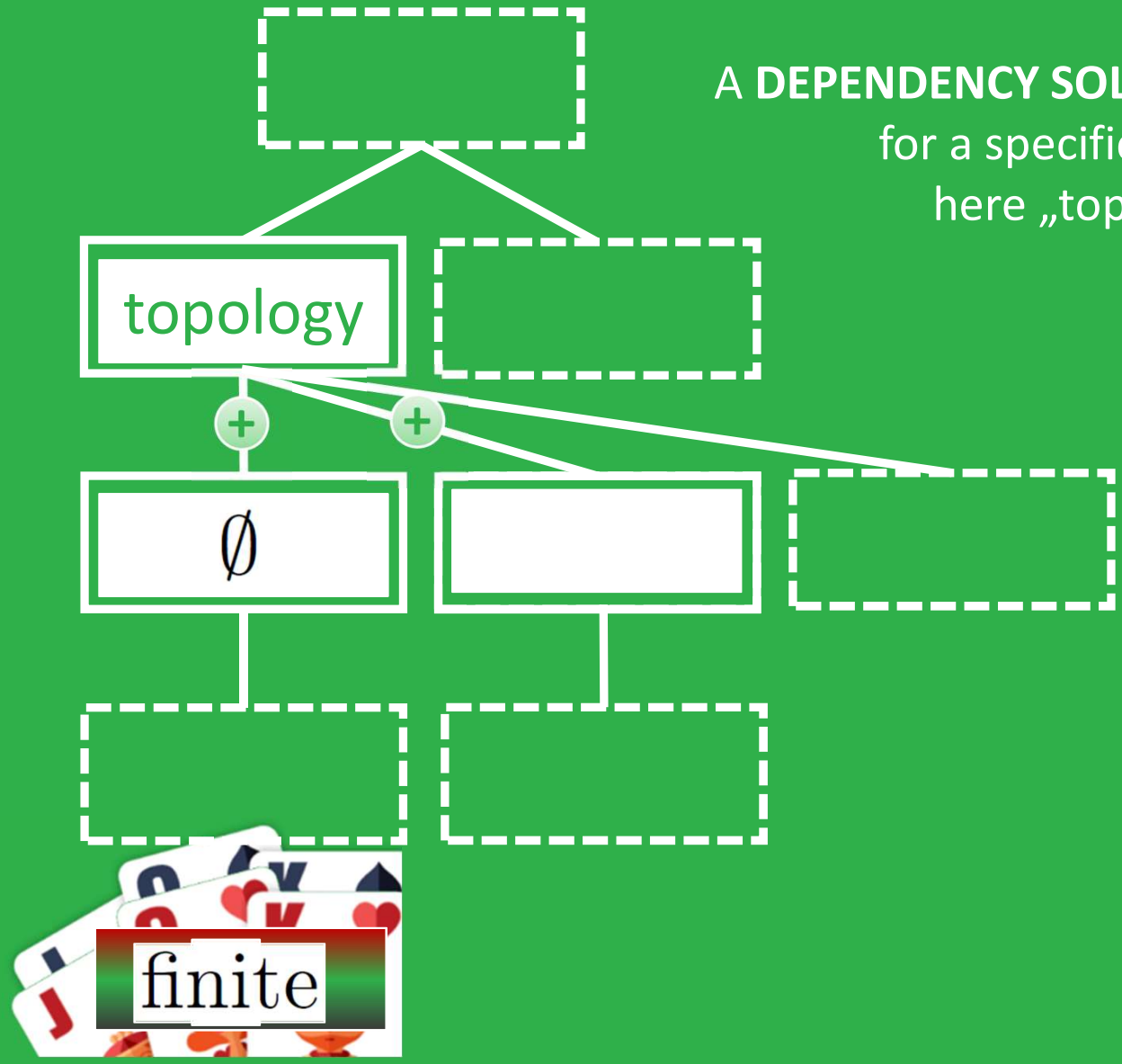
A Dependency Solitaire Game

- We have an order of „cards“ by dependencies:





A **DEPENDENCY SOLITAIRE**
for a specific term,
here „topology“



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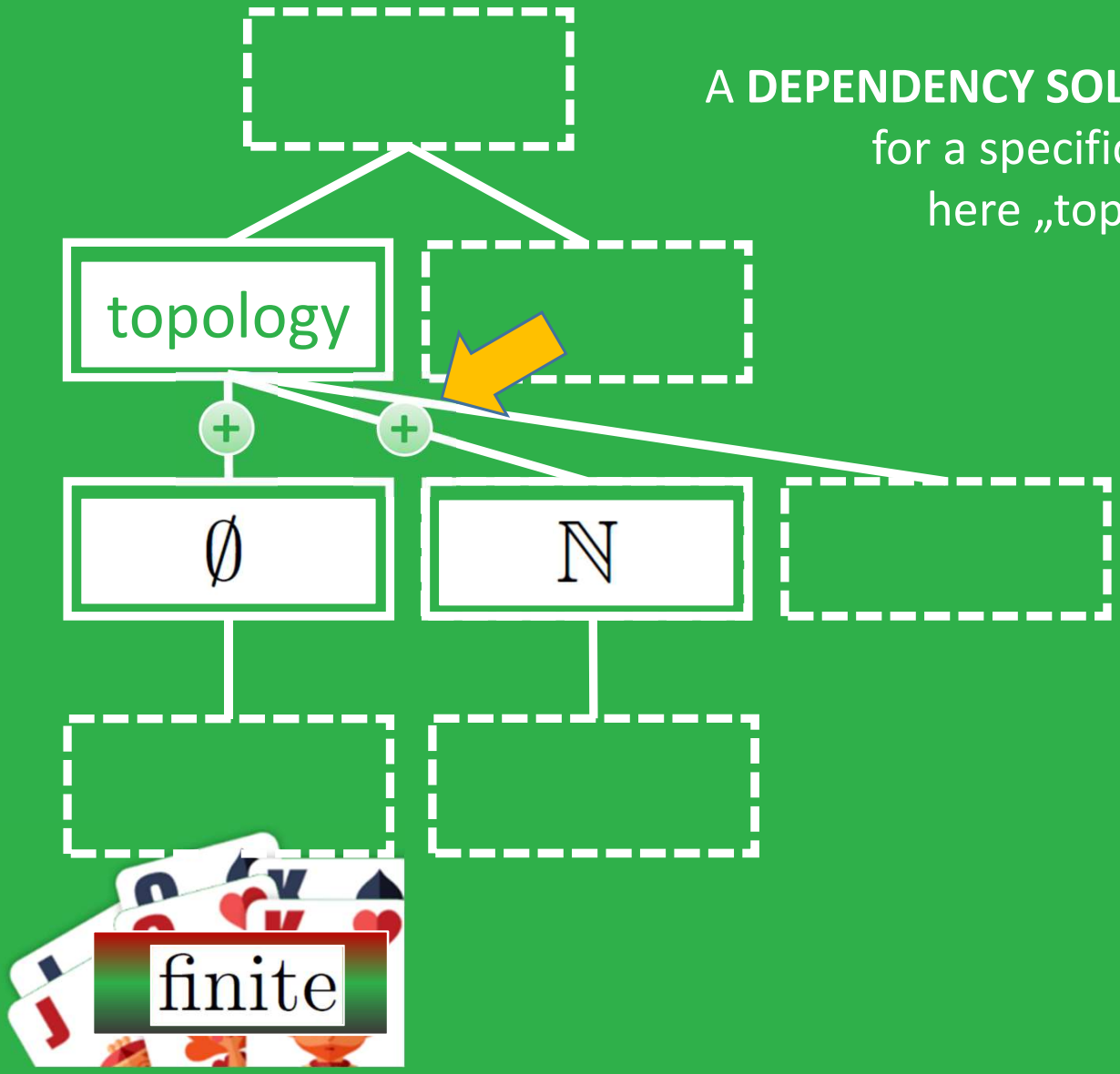

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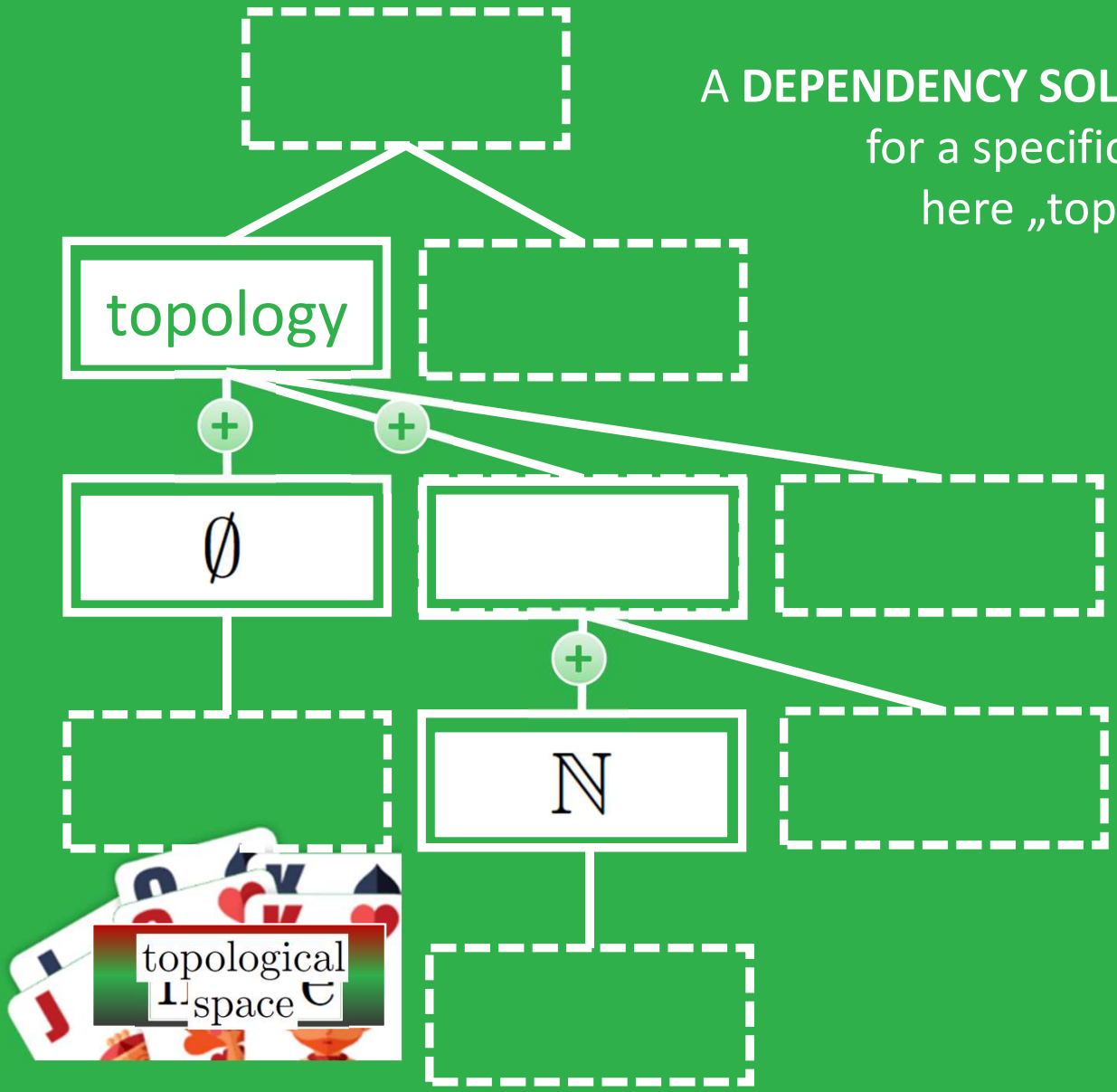
for a specific term,
here „topology“



SUMMARY
Intro to Eye Tracking for Math

- Research results for math
  - Math Literacy
  - Communities of Practice Visual Patterns
  - Math Competency
- We conducted an Eye Tracking experiment!
- Implications for Math

Hochschule für angewandte Wissenschaften Neu-Ulm

Neu-Ulm University of Applied Sciences

Wileystrasse 1
D-89231 Neu-Ulm

Phone +49 (0) 731-9762-0
Fax +49 (0) 731-9762-2299

info@hnu.de
www.hnu.de