Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA20111 (EuroProofNet)

Grantee name: Nathanael Arkor

Details of the STSM

Title: Towards 2-dimensional 2nd order algebraic theories

Start and end date: 10/03/2024 to 17/03/2024

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

The aim of the STSM was to catalyse work on presentations of 2-dimensional algebraic theories and – more speculatively – generalisations to other bases of enrichment, laying the groundwork for future work.

We began by studying enriched algebraic theories instantiated in the case where V = Cat. We then sketched a notion of presentation and discussed notions of presentation that might correspond to different structures on the induced 2-monad.

Because we wanted to make the most of our time in the same location, once we felt we had a rough understanding of the theory in this case – at least, to the extent that we could focus on the details while working remotely – we moved to the more general enriched case.

We considered various notions of enrichment that might be of use and settled upon enriching in the category of models for an essentially algebraic theory. This is general enough to permit many interesting examples, including those sketched in our STSM application, while also admitting a syntactic description. We therefore examined presentations of essentially algebraic theories, and developed ideas for multi-ary descriptions of these models. We also studied connections to the literature: although there does not seem to be a pre-existing "equational logic for essentially algebraic theories", we believe existing work can be adapted to cover them.

---

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

**Funded by
the European Union**

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

We believe the STSM lived up to its planned goals. We were able to work together at a whiteboard for long periods of time, clarifying the exact nature of the questions to be tackled and developing our own ideas for approaching them. As a result, we could begin to sketch answers to these problems. We will be working on these aspects remotely following the STSM, and have meetings already scheduled for this.

Our continuing work together will focus on the following areas.

(1) Making precise the notion of equational logic for essentially algebraic theories.
(2) Presentations for 2-monads, with applications to rewriting as described in our application.
(3) Presentations for theories enriched in a closed locally presentable category (= the category of models for an essentially algebraic theory).

In each case we are interested in studying (a) the appropriate monad-theory correspondence; (b) multi-ary models for the presentation; (c) interactions with the category-theoretic description of programming languages with novel features.

These build on the ideas we sketched during the STSM; indeed, some of the work involved is making precise ideas we sketched on a whiteboard and filling in the gaps we left in proofs while discussing. We envisage 2 – 3 papers based on this work.