

Report on the outcomes of a Short-Term Scientific Mission¹

Action number: CA20111 – European Research Network on Formal Proofs

Grantee name: Nathanael Arkor

Details of the STSM

Title: Modern algebraic perspectives on the λ -calculus and its extensions

Start and end date: 11/08/2024 to 18/08/2024

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

The STSM involved intensive discussion about the algebraic understanding of pure λ -calculus, between the grantee (Nathanael Arkor), the host (Martin Hyland), and a small group of other researchers in the areas of programming language theory, type theory, and category theory (Vikraman Choudhury, Dylan McDermott, and Philip Saville). We closely followed the work plan outlined in the proposal for the STSM, having a discussion with Martin each morning, followed by discussion with each other in the afternoons while Martin rested.

The STSM began with Martin explaining his current view of the contents of two of his papers on a modern algebraic perspective on the pure λ -calculus [1,2]. Analysing some of the proofs in those papers led to a discussion of combinators for the λ -calculus, in which Martin helped us to understand some of the existing work on the connections between λ -calculus and combinatory logic. We determined that this was the most important aspect of the λ -calculus that does not yet have a modern understanding, and so we decided to focus our attention on this. We successfully recast the connection between the SK-calculus and the λ -calculus in the context of Martin's work, and in doing so arrived at a deeper conceptual understanding of this connection than has been achieved previously.

We also spent some time during the week discussing other aspects of λ -calculus. In particular, we discussed models of λ -calculus arising from a notion of λ -monoid, the connection with closed categories, and the enriched structure of categories of models, making some progress in all of these. We also discussed how to repeat some of this work for variants of λ -calculus, such as for the linear λ -

¹ This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

calculus and for the computational λ -calculus. However, we determined that completing the remaining work on pure λ -calculus should be our priority in the short-term.

[1] Martin Hyland. 'Towards a notion of lambda monoid'. In: Electronic Notes in Theoretical Computer Science 303 (2014), pp. 59–77. DOI: 10.1016/j.entcs.2014.02.004.

[2] Martin Hyland. 'Classical lambda calculus in modern dress'. In: Mathematical Structures in Computer Science 27.5 (2017), pp. 762–781. DOI: 10.1017/S0960129515000377.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

The STSM achieved its planned goals. By discussing with Martin and between ourselves, we all gained a deeper understanding of the classical theory of the λ -calculus, and of Martin's more recent work. The STSM is the beginning of a collaboration between researchers across Europe, in which we will study the λ -calculus and its variants from an algebraic perspective. The results of this collaboration will contribute to type theory research in general. In the context of EuroProofNet, it will specifically contribute to Deliverable D4 ("Definition of a mathematical framework for modular reasoning about type theories and their extensions") and Research Coordination Objective 7 ("Develop a modular theory of type theories").

The main technical achievement of the STSM was in gaining a conceptual understanding of the connection between combinatory logic and the λ -calculus: in particular, the fundamental result that the combinators S and K provide a complete notion of computation. Understanding this connection was a specific goal of the STSM, and we achieved this goal completely. We intend to write a paper on this work in the near future and submit to CS conference (LICS or POPL), and then follow this with a more detailed journal article.

We intend to have a second in-person meeting in the following months to discuss this work further, which we expect may lead to subsequent publications.