

Report on the outcomes of a Short-Term Scientific Mission¹

Action number:

Grantee name:

Details of the STSM

Title: New axiomatization of synthetic differential geometry

Start and end date: 23/06/2024 to 29/06/2024

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

Work on Goals (1) and (2) was carried out, essentially as planned – alas with no clear result on, but suggestions for a local choice axiom.

For goal (1), we first had to figure out a good definition of open subsets and a good duality axiom which allows helps with the reasoning on these open subsets. We settled for a pointwise, synthetic topology-minded definition of open subset. We cleared up the relation to more traditional notions of open subset in SDG and computed how it relates to more local and global definitions we came up with.

To get a better duality axiom (better than the usual Kock-Lawvere axiom) we considered not the usual Weil-Algebras (at least not directly), but used instead the (easily internally formulated) endomorphism-theory of the real-line. This allowed us to propose a more general duality-axiom which should imply the Kock-Lawvere axiom.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

¹ This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

(max. 500 words)

Unexpectedly, during the work on goal (1), we established that our proposed, new duality axiom implies the field property which is usually postulated in SDG. From all our proposals for a local choice axiom, it is possible to derive the covering principle, provided one does not formulate it with metrically open sets. These are two unexpected minimizations of the axioms of SDG. Furthermore, we established with our axioms, that the real line is connected. Constructing function from our local-choice axioms which is common practice in SAG, is not yet possible in SDG, since we lack a good principle for partition of unity arguments. Due to this problem, we worked a bit more in other directions and did not try to semantically verify local choice axioms, since it is not clear yet which formulations are useful.

Goal (2) can be considered as completely solved. With the proposed duality axiom, the definition of formally étale types and maps carries over from SAG. We were able to show that finite types are formally étale with this definition, which allows to give many interesting examples of formally étale maps.

Our plan is to write down our findings and extend them to a publication.