

Short-Term Scientific Mission Grant - APPLICATION FORM¹ -

Action number: CA20111

Applicant name: Horatiu Cheval

Details of the STSM

Title: Proof mining libraries for Lean

Start and end date: 11/05/2025 to 17/05/2025

Detail of the cost in EUROS:

- Transport (upload screen capture): 237.6
- Hotel/day (upload screen capture): 187.92
- Food/day: 50

TOTAL: 1715.09

Goals of the STSM

Purpose and summary of the STSM.

(max.200 word)

The research program of proof mining seeks to analyze mathematical proofs with techniques from proof theory in order to extract additional information from them, such as computational content. Recent years have seen significant advances in libraries of formal mathematical proofs, for example in Lean's library, mathlib. Both Thomas Powell and I have an interest in proof mining, as well as in Lean, and there is a potential interplay between the two fields. On the one hand, the proof theoretical instruments from proof mining could be applied to existing formalized proofs from the mathlib library, in order to extract computational content from them. On the other hand, the kind of quantitative mathematical results typically coming out of proof mining are not usually encountered in libraries of formalized mathematics. The main goal of the STSM is to start a collaboration on these topics.

Working Plan

Description of the work to be carried out by the applicant.

(max.500 word)

We will discuss ideas on creating and organizing a library of formal mathematics for proof mining. At first, this will include lemmas on quantitative convergence results for sequences of real numbers satisfying certain recursive inequalities, which are at the basis of many of the results from the field. Other things we will consider are the API organization of particular types of spaces used in proof

¹ This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via-e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

mining, as well as formalization of concrete quantitative theorems on them. We will also investigate the extent to which we could build tools that help with the construction and usability of such libraries.

Furthermore, we will look at implementing tools like proof interpretations and general metatheorems for bound extraction in Lean, and how to approach integrating them with mathlib, as program extraction tools.

Starting points for our work could be implementations previously done by the applicant:

- <https://github.com/hcheval/tikhonov-mann>
- <https://github.com/hcheval/formalized-proof-mining>

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (<https://europroofnet.github.io/objectives/>) and deliverables (<https://europroofnet.github.io/deliverables/>).

Working groups to which this mission contributes: WG4

(max.500 words)

Beyond beginning a collaboration on Lean and proof mining, concrete outputs that are expected include:

- a library of formalized quantitative convergence lemmas for sequences of real numbers satisfying recursive inequalities, which are of crucial importance for proof mining, with potential automations for them;
- a library for mathematical structures commonly occurring in proof mining;
- an implementation of program extraction tools based on methods from proof mining, to be integrated with the mathlib library.

This ties into the WG4: Libraries of formal proofs and into Capacity Building Objective 5: Actively support young researchers, the under-represented gender, and teams from regions with less capacity.