Short-Term Scientific Mission Grant
- APPLICATION FORM[1] -

Action number: CA20111 - European Research Network on Formal Proofs

Applicant name: Vikraman Choudhury

Details of the STSM

Title: Modern algebraic perspectives on dualities and control effects

Start and end date: 01/28/2025 to 02/28/2025

Detail of the cost in EUROS: 3200

As reference, you can use the daily allowances by country for ITCGs (https://europroofnet.github.io/itcg-daily-allowance/) and the associated Excel sheet (https://europroofnet.github.io/_pages/grant.xlsx).

- Transport (upload screen capture): NA

- Hotel/day (upload screen capture): 72 GBP = 87 EUR per night for 32 nights

- Food/day: NA

TOTAL: 3200 EUR

Goals of the STSM

Purpose and summary of the STSM.

(max.200 word)

Continuations are an important concept in computer science. In functional programming, continuations allow programmers to manipulate the control flow of programs, using control operators. In compiler design, continuation-passing-style is used as an intermediate representation for code generation and compiler optimizations. In denotational semantics, continuations are used to understand the evaluation order of programs, monadic effects, effects and handlers. Continuations also give a computational interpretation of classical logic. A unifying view of continuations can be obtained by studying them using category theory and duality, using double dualization or continuation monads (Kock), categories of continuations (Hofmann and Streicher), tensor-not categories (Thielecke), control categories (Selinger), C-categories (Führmann and Thielecke). This was an active research area about 25 years ago.

Very recently, the applicant (Vikraman Choudhury) has revisited the categorical structure of continuations, from the point of view of dual adjunctions (closure and coclosure), and produced a language (λλ~ calculus) which exhibits a new duality of abstraction and coabstraction, using it to explain different computational perspectives on continuations, control effects, and classical logic (to appear at POPL 2025). The host (Prof. Marcelo Fiore) has another modern perspective on continuations and control effects, called Inception algebras, which is derived from the modern understanding of universal algebra: Lawvere theories and Substitution algebras. The purpose of this STSM is to connect these two lines of research.

---

[1] This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

**Funded by
the European Union**

[1] Kock, Anders. "On double dualization monads." Mathematica Scandinavica 27.2 (1970): 151-165.

[2] Hofmann, Martin, and Thomas Streicher. "Continuation models are universal for/spl lambda//sub/spl mu//-calculus." Proceedings of Twelfth Annual IEEE Symposium on Logic in Computer Science. IEEE, 1997.

[3] Thielecke, Hayo. "Categorical structure of continuation passing style." (1997).

[4] Selinger, Peter. "Control categories and duality: on the categorical semantics of the lambda-mu calculus." Mathematical structures in computer science 11.2 (2001): 207-260.

[5] Führmann, Carsten, and Hayo Thielecke. "On the call-by-value CPS transform and its semantics." Information and Computation 188.2 (2004): 241-283.

[6] Choudhury, Vikraman, and Gay, Simon J. "The Duality of λ-Abstraction." To appear in Principles of Programming Languages (2025).

[7] Fiore, Marcelo. "Algebraic Theories and Control Effects, Back and Forth." *Talk given at the Semantics of Proofs and Programs Workshop. uRl: https://www. cl. cam. ac. uk/~ mpf23/talks/ihp2014. pdf (cit. on p. 26)*.

Working Plan

Description of the work to be carried out by the applicant.

(max.500 word)

Practical aspects

The applicant (Vikraman) plans to visit the host (Marcelo) at the Computer Laboratory, University of Cambridge. The applicant and host have previously collaborated on various topics related to type theory and category theory, and Vikraman has been a long-term visitor to the Computer Lab as part of the CLASH group, as a PhD student on a Paul Purdom fellowship, and as a postdoc on an (ongoing) Marie-Skłodowska-Curie fellowship (ReGraDe-CS, no. 101106046). Vikraman will be partially supported by the fellowship, hence we are only asking for support for accommodation in Cambridge for a month. Gregor Feierabend, who is a PhD student of Marcelo supported by a Gates Scholarship, will also be joining the discussions.

Our plan is to first, understand the concrete categorical structure of inception algebras developed by Marcelo (since it only exists as a talk and is not published in a paper), then connect it to the other lines of research on continuations and duality.

Technical aspects

Concretely, we would like to tackle 3 different research questions.

1. Inception algebras are derived abstractly from Lawvere theories and Substitution algebras, which comes from the research programme on Abstract Syntax, developed by Marcelo and collaborators over the last two decades. The semantics of inception algebras uses a strong monad on a functor category [F,Set], and it validates the usual control effects and non-local jumping behaviour of continuations, which are otherwise understood using the double negation or continuation monad. Other categorical axiomatizations of continuations are all known to be equivalent to Kleisli categories (or their duals) of continuation monads. It must be understood how these two models are related, by constructing functors between them and understanding their abstract properties.
2. In Vikraman's work on λλ~, a conceptual understanding of the negation functor acting on an arbitrary category is given, and by starting from cartesian closure with coproducts, one can derive cocartesian coclosure, which leads to the syntax of the λλ~ calculus. The next step is to investigate what happens when one adds additional structure such as fixpoints, by requiring

the base category to be some category of domains, or CPO-enriched (such as in Marcelo's FPC calculus), understand how fixpoints interact with continuations and control operators by reading off the equational theory, and prove an adequacy theorem. Marcelo's expertise in domain theory is extremely relevant to tackle such a question.

3. Due to the non-local jumping behaviour of continuations, the question of normalization for λ-calculi with control operators, or dual calculi such as λμ, is a hard problem to solve. It is known that λμ with a subset of beta-reductions is normalizing, but this uses low-level proof-theoretic and reduction-based arguments. A modern algebraic treatment using normalization-by-evaluation arguments is missing. Vikraman's work on λλ~ presents a better dual calculus (only call-by-value), because the syntax follows standard type-theoretic presentation (one-sided sequents, with natural deduction rules, or multicategorical), with an equational theory. Following the NbE understanding of simply-typed λ-calculus (without or with sums), this is a better fit for NbE frameworks, which are Marcelo's expertise. Our plan is to develop NbE for λλ~, by extending the usual gluing models with dual adjunctions (to interpret λ~), starting from the Kripke semantics of classical logic.

[1] Fiore, Marcelo, Gordon Plotkin, and Daniele Turi. "Abstract syntax and variable binding." Proceedings. 14th Symposium on Logic in Computer Science (Cat. No. PR00158). IEEE, 1999.

[2] Fiore, Marcelo. "Second-order and dependently-sorted abstract syntax." 2008 23rd Annual IEEE Symposium on Logic in Computer Science. IEEE, 2008.

[3] Fiore, Marcelo, and Ola Mahmoud. "Second-order algebraic theories." Mathematical Foundations of Computer Science 2010: 35th International Symposium, MFCS 2010, Brno, Czech Republic, August 23-27, 2010. Proceedings 35. Springer Berlin Heidelberg, 2010.

[4] Fiore, Marcelo, and Dmitrij Szamozvancev. "Formal metatheory of second-order abstract syntax." Proceedings of the ACM on Programming Languages 6.POPL (2022): 1-29.

[5] Fiore, Marcelo P. "Axiomatic Domain Theory." (1996).

[6] Fiore, Marcelo. "Semantic analysis of normalisation by evaluation for typed lambda calculus." Proceedings of the 4th ACM SIGPLAN international conference on Principles and practice of declarative programming. 2002.

[7] Balat, Vincent, Roberto Di Cosmo, and Marcelo Fiore. "Extensional normalisation and type-directed partial evaluation for typed lambda calculus with sums." ACM SIGPLAN Notices 39.1 (2004): 64-76.

[8] Ilik, Danko, Gyesik Lee, and Hugo Herbelin. "Kripke models for classical logic." Annals of Pure and Applied Logic 161.11 (2010): 1367-1378.

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (https://europroofnet.github.io/objectives/) and deliverables (https://europroofnet.github.io/deliverables/).

Working groups to which this mission contributes:

(max.500 words)

The topic of this work is in fundamental issues related to the (typed) λ-calculus and its (dual) extensions, which is the basis of type theories, hence this work falls within the domain of the working group WG6: Type Theory. The categorical semantics of (dual) calculi and research on their metatheory is relevant to the Deliverable D4: "Definition of a mathematical framework for modular reasoning about type theories and their extensions", and Research Coordination Objective 4: "Develop a modular theory of type theories".

We plan to disseminate the insights and results we obtain during the research visit and collaboration, in the form of talks and open-access research papers submitted to relevant conferences in logic and

theoretical computer science (such as, ETAPS, POPL, LICS). This contributes to Capacity Building Objective 8: "Disseminate the results of the Action activities to the scientific community, the industry, the certification bodies, the European institutions and to the general public".

To point out an important communal issue about why this research is important, we note that the subject of continuations and duality has a very fragmented viewpoint in the research community, which is detrimental to the social process of peer-reviewing, and making real-world impact. The American point of view on continuations is via control operators which reify evaluation contexts, while the French-Italian point of view on continuations is via term calculi for classical logic, or the duality of programs and contexts. A mathematical understanding of both points of view using modern algebraic tools reveals the fact that they are manifestations of the same categorical structure, dressed up in different ways. The applicant (Vikraman)'s work on $\lambda\lambda\sim$ makes this very explicit: the $\lambda\sim$ is a dual binder, like μ, but is also a generalized control operator, like callcc. We believe that developing this viewpoint further is important to the objective of promoting integration between various research circles, and hence relevant to Capacity Building Objective 1: "Bring together members of the different communities working on proofs in Europe", and Objective 6: "Transfer knowledge in terms of expertise, scientific tools and human resources across the different disciplines and between academia and industry".

Finally, Vikraman is an early-career researcher, who has had an academic career so far in three different regions, namely North America, United Kingdom, and Europe, which makes them uniquely suited to carry out this research programme, that requires multiple different points of view. This contributes to the Capacity Building Objective 5: "Actively support young researchers, the under-represented gender, and teams from regions with less capacity".