Short-Term Scientific Mission Grant

• APPLICATION FORM[1] -

Action number: CA20111

Applicant name: Maximilian Doré

Details of the STSM

Title: Proof automation in Cubical Agda

Start and end date: 13/02/2023 to 17/02/2023

Goals of the STSM

Purpose and summary of the STSM.

(max.200 word)

The goal of the STSM is to understand proof search in cubical type theories from a complexity-theoretic point of view and use these insights to devise efficient tactics for theorem provers such as Cubical Agda.

Logics inspired by higher-dimensional structures such as cubical sets give a principled way to reason about coherences, but are quite difficult to work in. By automating proof search in such a logic, we hope to further the understanding of such logics and make them more usable for researchers working on formalisations in constructive type theories with equality.

The idea for the mission came about during the WG6 kick-off meeting in May 2022 where I presented very early work on a tactic for Cubical Agda. After discussions with Anders Mörtberg and Evan Cavallo the idea emerged to use a different characterisation of cells in a cubical set than is used in Cubical Agda, namely that of cells as certain poset morphisms. This point of view has suggested a significantly more efficient method for finding cells with a certain boundary, which we now hope to exploit to attain novel proofs in Cubical Agda.

Working Plan

Description of the work to be carried out by the applicant.

(max.500 word)

We have come up with a basic working principle for the theorem prover, at the STSM we want to refine this and use it in practice. More concretely, we plan to work on the following subprojects:

[1] This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

- Include path reversals in proof search: Many proofs in Dedekind cubes which require Kan compositions can be simplified if the cubical sets support path reversals. Cubical Agda is built on De Morgan cubes which do support path reversals; hence we can simplify many proof terms of our solver if it uses path reversals if necessary. Since poset maps on partially specified boundaries with can be used to come up with interval substitutions with reversals, we want to broaden our approach to also find all interval substitutions in De Morgan cubes.
- Use admissible rules to simplify proof terms: There are common constructions such as cube fillers which appear in many cubical proofs. If the tactic realises that a part of a proof can be constructed by using such a "lemma", it would be good to use an instantiation of such lemma in a proof instead of a heavily nested Kan composition.
- We want to use the tactic to devise novel proofs. For instance, the pentagon for the smash product has so far evaded formalisation in HoTT/Cubical Agda, we hope that with the tactic we are able to remedy that.
- Connect with dependent type theories: If we have parameters in constructors of a HIT, we need to connect the cubical proof search with term synthesis in dependent type theories. Using a congruence closure mechanism, we hope to make the tactic usable in many more practical examples.
- Integrate the tactic in Cubical Agda: At the moment, there exists a prototype for the tactic in Haskell (https://github.com/maxdore/cubetac). Even though external programs can be called from a tactic, the current setup means that an external Haskell program needs to be installed. For usability it would be beneficial to reimplement at least a simplified version of the solver directly in Agda and make it part of the Cubical library.

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (either research coordination and/or capacity building objectives) and deliverables.

(max.500 words)

We plan to have the provide the following deliverables after the STSM:

- We want to submit the results of our complexity-theoretic considerations and descriptions of the algorithms to a conference, possible the Conference on Automated Deduction 2023. We propose cell search in cubical sets as a novel problem for automated proof search.
- We will aim to construct novel proofs of results which could so far not be formalised in either HoTT or Cubical Agda. Thereby we want to grow the Cubical library.
- We will provide an implementation of the tactic in Cubical Agda which can be used by any user of Cubical Agda. Coming up with Kan compositions and interval substitutions is a daunting task for newcomers to Cubical Agda, by providing a tactic we make it easier to use Cubical Agda in practice. This will hopefully spark more formalisations and grow the Cubical library further.

By providing a tactic for cubical type theories we hope to contribute directly to *boosting the usability of proof systems*, the second tenet of the COST action. The project has grown out of the kick-off meeting of WG 6 and is most closely linked to the syntax type theories, but also builds a bridge to other working groups: We are proposing cell search in cubical sets as a novel problem for automated theorem provers (WG 2) and provide first solutions for this problem. We hope to add more proofs the Cubical library, thereby contributing to libraries of formal proofs (WG 4). After the STSM we hope to connect with these two working groups and use their insights to carry forward this project.