# Report on the outcomes of a Short-Term Scientific Mission[1]

**Action number: CA20111**

**Grantee name: Maximilian Doré**

## Details of the STSM

Title: **Proof automation in Cubical Agda**

Start and end date: 27/02/2023 to 03/03/2023

## Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

*(max. 500 words)*

In Stockholm I worked with Anders Mörtberg and Evan Cavallo on a tactic for Cubical Agda, which solves new kinds of proof goals introduced by Cubical Agda. We spend the days discussing and on March 1 I gave a talk at the *Computational Mathematics Seminar* to get feedback from other researchers at Stockholm University, in particular some combinatoricists who are working on graph-theoretic problems.

We have worked both on improving the theoretical understanding of the algorithmic problems, and on extending the scope of the tactic in practice by trying out different proof heuristics for finding proofs. In order to construct a cell with a given boundary, Cubical Agda has introduced two novel reasoning principles:

1. Contortions: Lower-dimensional cells given as constructors of a higher inductive type can be transformed into higher-dimensional cells with interval substitutions.
2. Kan composition: One can construct an open box whose composition will then give the required cell.

We have worked on both of these reasoning principles. One of these researchers at the seminar talk, Marc Hellmuth, suggested to establish NP-hardness for problem of finding contortions in cubical sets. This was a very natural idea which we hadn't thought about before. We are currently working on a reduction from satisfiability of monotonous Boolean formulas to finding cells in a cubical set. Thereby we

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

contribute to the complexity-theoretic study of this problem, which is decidable as there are only finitely many contortions of a cell in a given dimension.

For Kan compositions, the composed box can have sides which are themselves the result of Kan compositions. The search space hence grows infinitely large. It is very natural that the problem of finding Kan compositions is undecidable, we have been able to establish this with a reduction from string rewrite systems. To still make the problem tractable, we have discussed several heuristics that can be used to still find proofs in many instances.

We have decided to not work on incorporating reversals for the time being as the Dedekind cubes also give rise to nice proofs.

## Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

*(max. 500 words)*

The STSM was very successful, and we have achieved our goal of making the tactic more powerful.

I will present results of the STSM at meeting of WG6 of the EPN network in Vienna in April 2023. We have decided to not submit a paper to the Conference on Automated Deduction 2023 and instead plan to submit a paper to the Journal for Automated Reasoning (JAR) in the coming months. The journal does not have a page limit and hence allows to present our work in more detail.

Another deliverable is the tactic for Cubical Agda which we plan to make part of either Agda or the Cubical Agda library.

We have hence contributed to the objective of boosting the usability of proof systems, the second tenet of the COST action. More specifically, we have contributed to the different work groups as follows:

- By exploring a different notion of cubical sets based on monotonous poset maps we have furthered the understanding of syntax and semantics of type theory, which is the goal of WG 6.
- We are establishing the proof obligations of Cubical Agda as a novel problem for automated theorem provers (WG 2). To understand the current literature on cubical type theory one has to collect develop substantial expertise, by writing an article for the JAR we hope to make cubical type theory more accessible to researchers in the automated theorem proving community and hope to initiate more research on effective algorithms for finding proofs in cubical type theory.
- As the tactic finds proofs which are often quite different from what a human prover came up with, we have constructed novel proofs which we plan to include the Cubical library, thereby contributing to libraries of formal proofs (WG 4)

We are currently continuing the collaboration remotely.