

Report on the outcomes of a Short-Term Scientific Mission¹

Action number: CA20111
E-COST-GRANT-CA20111-f0b0e7f3

Grantee name: Maribel Fernandez

Details of the STSM

Title: Hierarchical Higher-Order Port Graphs for the representation and analysis of proofs

Start and end date: 11/092022 to 19/09/2022

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

As mentioned in the application, our long-term goal is to develop a graph-based tool for the representation, analysis and management of proofs using graph rewriting techniques. As a first step, we aim to develop representations of proofs using hierarchical port graphs, which we hope can serve as a common language for the encoding of proofs generated by different proof systems. For this, according to the proposed work plan, we checked the suitability of hierarchical port graphs and rewriting rules as a modelling tool for proof systems, by using them to specify proofs in intuitionistic logic. We also considered the use of hierarchical port graph rewriting rules to specify operations on proofs (used in proof normalisation) and considered strategies to control the rules to achieve uniqueness of normal forms. More specifically, we addressed the following tasks mentioned in the work plan:

1. A comparison of existing graph-based proof representation languages, focusing on the features of graphs used in the encoding of proofs. We reviewed existing work on graph-based proof representations and summarised their main features. This work contributes towards the Action objectives, in particular Objective 8 (develop the use of natural or controlled languages in proof systems) since graph-based representations of proofs can suggest new graphical languages for proof systems.

¹This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

2. We defined encodings of proofs in intuitionistic logic using hierarchical higher-order port graphs. We plan to continue working on the encodings and complete them in the following months (this will be reported in a paper that we have started writing). This will also contribute towards Action Objective 8.
3. We considered the suitability of graph representations of proofs as a tool to communicate and exchange proofs between different proof assistants, and as a visual tool to facilitate proof construction, extension and update. The latter is relevant to the work done in WG4 (Libraries of formal proofs), whose aim is to investigate approaches to efficiently maintain libraries of proofs, so that they can be modified and queried by users without expert knowledge of the system used to develop proofs.
4. Based on the encodings of proofs of intuitionistic logic using graphs, we started the process of identification of the main features required for a general graph-based language for proof representation and manipulation. This is a first step towards the design of a domain-specific version of a graph-based modelling tool, such as PORGY, to model/analyse proofs.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

The visit achieved the planned goals and outcomes. We are currently working on a journal article describing intuitionistic-proof representations using hierarchical higher-order port graphs (this corresponds to Tasks 1 and 2 in the work plan given above). We expect this will be submitted for publication in 2023. We have already written preliminary versions of the following sections:

- Related work: summarising and comparing our approach with related approaches using for example bigraphs - Task 1 in the work plan above.
- Representation of proofs from intuitionistic logic using hierarchical higher-order port graphs (HoP) - Task 2 in the work plan above.
- Characterisation of the HoP that correspond to proofs - Task 2 in the work plan above.
- Some simplification rules for proofs - Task 2 in the work plan above.

In future, we will complete the study of the representation and simplification of intuitionistic logic proofs using graph rewriting rules, and we will then focus on the representation of proofs in powerful logical frameworks such as LF modulo, which is the basis of Dedukti. We will then aim to design a domain-specific language for proof representation. More specifically, we plan to continue this work to develop the foundations for a graph-based proof management environment (in the style of PORGY but specifically tailored to the management of proofs). This will require some engineering work as well as more research on proof formats and proof management. This research can also shed light on closely related problems, such as, proof visualisation, proof search, proof maintenance (e.g. proof updates and extensions).

To carry out this work, we are planning a follow up visit of Sandra Alves to London in November 2023 to complete the specification of graph transformation rules implementing

proof simplification. In addition, we are planning to involve undergraduate and postgraduate students from the University of Porto and King's College London in this collaborative work: during the visit, we discussed potential topics for final-year BSc and MSc projects that combine theoretical work with implementation towards the above-stated goals. As a result, we have proposed student projects (for the 2022-23 academic year) in topics related to this visit, both in the university of Porto and at King's College London.