Short-Term Scientific Mission Grant
- APPLICATION FORM[1] -

Action number: COST Action CA20111 EuroProofNet

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Details of the STSM

Title: Quantitative Types for Programming Languages with Global State

Start and end date: 03/07/2022 to 08/07/2022

Goals of the STSM

Applicant enters max. 200 word summary here.

The **call-by-push-value** (**CBPV**) paradigm distinguishes between values and computations by introducing two primitives *thunk* and *force*. The former turns a computation into a value, the latter turns a value into a computation. Thus, different reduction strategies such as call-by-name and call-by-value are captured by CBPV by conveniently labeling a program using *force* and *thunk*.

In most programming languages, variables having global scope are accessible from any place of the program. The set of all global variables of a program is known as the **global state**. In some languages, all variables are global, while in pure functional languages variables have limited scope, though global variables are often available by declaring a variable at the top level of the program.

The aim of this project is to define a **quantitative type system** for an extension of the **CBPV** paradigm including **global state** constructors. The type system is supposed to give a quantitative semantics to the language, so that we will be able to study and measure quantitative properties of call-by-push-value with global state by means of its associated quantitative system. We are interested in properties related to both time (evaluation steps) and use of memory (locations on global state).

Working Plan

Applicant enters max. 500 word summary here.

In order to succeed in our project, we plan to decompose the work on two well distinguished steps: the design of the untyped language following a call-by-push-value paradigm with global state, and the design of the quantitative type system, following the non-idempotent intersection type paradigm.

[1] This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

The first objective of our planning work would be to define an untyped syntax for the call-by-push-value language with global state, together with an appropriate small step operational semantics. Thisimplies in particular the characterization of the notion of normal form, by means of an appropriate grammar and a sound and complete characterization theorem.

A second objective is to understand the notion of clash in the new language, which gives an intuition of the untyped terms/programs that are going to be discarded by any reasonable type system. This also implies the characterization of the notion of clash-free normal form, by means of an appropriate grammar and a second sound and complete characterization theorem.

Types will be then added to the untyped language with global states. This is a third objective. We will first study simple types, which are natural, simple, and intuitive. But then we will add intersection types, and in particular, quantitative intersection types, which are able to capture quantitative properties of the operational semantics of the language by replacing the standard notion of idempotent intersection by the non-idempotent one. This amounts to replace sets of types by multiset of types.

A fifth objective of the project, the more ambitious one, is to prove quantitative properties of the language by means of the quantitative type system defined in the previous point. Typically, one expects to be able to show quantitative subject reduction and quantitative subject expansion, together with appropriate lemmas for substitution and anti-substituton. Soundness and completeness of the method should be possible to derive, where quantitative soundness reads as “If a term is typable by a derivation of size n, then t can be evaluated in at most n evaluation steps”. Completeness of the approach would be the last objective. Refinement of upper bounds to exact measures should also be studied.

Expected outputs and contribution to the Action MoU objectives and deliverables.

Quantitative type-systems have been successfully used in several recent resource aware consumption investigations in the realm of evaluation strategies implemented by different functional programming languages and/or modern proof-assistants. They allow for the extraction of exact bounds for these evaluation strategies, thus providing concrete tools to study quantitative properties (times and space) of these systems.

To the best of our knowledge, no previous formalism for call-by-push-value and global states exists already in the literature. This is then an original line of research resulting in a novel model, which will be designed in particular to capture different functional reduction strategies such as CBN and CBV, as well as different reduction strategies using global state.

The quantitative study of properties related to global state will also be an advancement to the knowledge of computational resource analysis.
As mentioned before, the more ambitious goal is to prove quantitative properties of the call-by-push-value language with global states by means of a quantitative type system. We also expect to be able to refine the classical “upper bound” approach to à more ambitious goal pursuing “exact bounds” of computation.

As an expected deliverable, we expect to publish a conference paper on this topic.

The topics of this collaboration can be integrated in the WG3 and WG6.