

Report on the outcomes of a Short-Term Scientific Mission¹

Action number: CA20111

Grantee name: Delia Kesner

Details of the STSM

Title: Quantitative Types for Programming Languages with Global State Start and end date: 10/07/2022 to 15/07/2022

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

This STSM was dedicated to an ongoing work in progress between Sandra Alves, from the University of Porto, Miguel Ramos, a PhD student at the University of Porto under the supervision of Sandra Alves and myself, and myself (Universite Paris Cite).

The aim of our project is to define a quantitative type system for a programming language with global state. The typing system is inspired from intersection type theory, and it is supposed to be not only sound (with respect to termination), but also complete. This means that we look for a typing system which is a model, in the sense that the typing is invariant by reduction of terms. The main result that we would like to prove is that every typable term is normalizing, and every normalizing term is typable. Since the typing system is quantitative, we expect the typing system to give a decreasing measure to reduction, that is, when a term t is typable with some derivation Phi, and t reduces in one step to t'. then also t' is typable with some derivation Phi' such that the size of Phi is greater than the size of Phi'. This quantitative property is very strong, and allows in particular to prove soundness in a very simple way (no reducibility techniques would be necessary).

During the STSM we have first proved some properties about untyped terms and untyped normal forms in the programming language with global stages. We also started to define a typing system to capture quantitative properties of the language. Our types are of the form (i, T, f), where I and f represent states, and T is a (quantitative) type. When a term t has type (I, T, f) this means that t can be executed with initial state I, and the execution of t yields to a result of type T and a final state f. We have already proved some basic properties of the typing system, and we are currently exploring



¹This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.



subject reduction, which does not seem trivial.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

This STSM achieved its planned goals, because we succeded to have a clean characterization of untyped terms with "clashes", that play a crucial role in the typed study. Intuitively, not only non-terminating terms are going to be discarded by the typing system, but also terms with clashes should be discarded. Indeed, meaningless terms in this framework are caused by three main reasons, the first one is a syntactical one, the second one is related to the bad composition of a term with a state (the term tries to access the global state in an undefined position), and the third one is non-termination. Our work during this STSM allowed us to characterize the two former kind of "clashes". This was done by a context free grammar which was proved to have several properties.

The next step of our collaboration is to study the first quantitative typing system we have defined. Some typical properties (like spreading) have been already proved. We also have a good candidate for the notion of "size" for type derivations. However, we still need to understand how the type of a term, changing an initial to a final state, can be abstracted away in order to construct a (quantitative) model. We are currenly exploring an idea which seems to be promising.