Short-Term Scientific Mission Grant

- APPLICATION FORM[1] -

Action number: CA20111 – European Research Network on Formal Proofs

Applicant name: Dylan McDermott

Details of the STSM

Title: Modern algebraic perspectives on the λ-calculus and its extensions

Start and end date: 11/08/2024 to 18/08/2024

Goals of the STSM

Purpose and summary of the STSM.

(max.200 word)

The λ-calculus is a formalism for computation that provides the foundation for modern type theory, and underlies many current developments in the design and implementation of programming languages. The study of the λ-calculus as a mathematical object was the focus of lively research throughout the 20th century (see e.g. [1]). However, active research in pure λ-calculus has declined over the last twenty years, with prominent members of the community retiring. Consequently, a wealth of deep understanding is at risk of being forgotten, despite indications that it sheds light on modern lines of research. In the last decade, Martin Hyland (Professor Emeritus at the University of Cambridge) has employed ideas inspired by recent developments in the study of programming languages, universal algebra, and category theory to give insights into the algebra of the λ-calculus. However, only a small portion of this work has been published [2, 3].

Our objective is to gather a small group of experts in programming language theory and categorical logic together with Hyland for a week of intense collaboration, laying the foundations for an ongoing collaboration in which Hyland's research programme can be realised, and facilitating extensions of these ideas to variants of the λ-calculus relevant in modern programming languages (e.g. differentiable and probabilistic calculi [4, 5, 6]).

[1] Henk P. Barendregt. The lambda calculus: its syntax and semantics. Vol. 103. Studies in Logic and the Foundations of Mathematics). Revised edition. North-Holland, 1985. ISBN: 978-0444875082.

[2] Martin Hyland. 'Towards a notion of lambda monoid'. In: Electronic Notes in Theoretical Computer Science 303 (2014), pp. 59–77. DOI: 10.1016/j.entcs.2014.02.004.

[3] Martin Hyland. 'Classical lambda calculus in modern dress'. In: Mathematical Structures in

[1] This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via-e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

Computer Science 27.5 (2017), pp. 762–781. DOI: 10.1017/S0960129515000377.

[4] Sam Staton. ‘Commutative Semantics for Probabilistic Programming’. In: Programming Languages and Systems. Ed. by Hongseok Yang. Berlin, Heidelberg: Springer Berlin Heidelberg, 2017, pp. 855–879. ISBN: 978-3-662-54434-1.

[5] Chris Heunen, Ohad Kammar, Sam Staton and Hongseok Yang. ‘A convenient category for higher-order probability theory’. In: 2017 32nd Annual ACM/IEEE Symposium on Logic in Computer Science (LICS). 2017, pp. 1–12. DOI: 10.1109/LICS.2017.8005137.

[6] Matthijs Vákár. ‘Reverse AD at Higher Types: Pure, Principled and Denotationally Correct’. In: Lecture Notes in Computer Science. Springer International Publishing, 2021, pp. 607–634. isbn: 9783030720193. DOI: 10.1007/978-3-030-72019-3 22.3

Working Plan

Description of the work to be carried out by the applicant.

(max.500 word)

Practical aspect

The applicant (Dylan McDermott) will gather with the host (Martin Hyland) and a small group of researchers specialising in programming language theory, type theory, and category theory (Nathanael Arkor, Vikraman Choudhury, Philip Saville). In particular, the applicant and Arkor have previously collaborated in studying algebraic structures with variable-binding structures [7, 8], which builds upon aspects of Hyland’s work in [2, 3] above.

Our plan is to use the week to understand the fundamental ideas in Hyland’s planned research programme concerning the algebraic structure of the λ-calculus. Hyland has an extensive vision extending the initial perspective developed in [2, 3], but a lack of time and collaboration means these insights risk being lost. We expect to spend half of each day of the visit in discussion with Hyland; and the other half of each day in discussion with each other, developing our understanding and exploring potential applications to modern programming language theory, while allowing Hyland to rest.

[7] Nathanael Arkor, and Dylan McDermott. ‘Abstract Clones for Abstract Syntax.’ In: 6th International Conference on Formal Structures for Computation and Deduction (FSCD 2021). Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2021.

[8] Nathanael Arkor. ‘Monadic and Higher-Order Structure’. Doctoral thesis, University of Cambridge (2022).

Technical aspect

Hyland’s work in [2, 3] proposes that modern techniques in categorical logic may be used to understand the algebraic structure of the λ-calculus in a way that clarifies and generalises the classical understanding, which is grounded in the syntax-heavy techniques of formal logic. In particular, variable binding, the most fundamental aspect of the λ-calculus, admits an elegant treatment in terms of algebraic clones (closely related to algebraic theories, cartesian multicategories, and relative monads) equipped with algebraic structure (cf. [7, 8]). Hyland shows in [2, 3] that several fundamental results in the classical theory of the λ-calculus – for instance, the relationship between λ-theories and λ-algebras – may be recovered elegantly using standard category theoretic techniques.

In [2, 3], Hyland mentions several topics he had not been able to include in the papers – for instance, the connection to combinatory logic and the universal algebraic perspective on the λ-calculus – as well as avenues for future research from this perspective, such as the relationship with differential λ-calculus. Over the course of the week, we plan to discuss these topics, to understand precisely the connections to which Hyland alluded and to develop them further.

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (either research coordination and/or capacity building objectives) and deliverables.

(max.500 words)

The work proposed here will contribute to Working Group 6. Our aim ultimately is to disseminate the insights we develop during and following the research visit in Cambridge in the form of papers submitted to relevant computer science conferences (e.g. ETAPS, LICS, POPL). This contributes to Capacity Building Objective 8 ("Disseminate the results of the Action activities to the scientific community, the industry, the certification bodies, the European institutions and to the general public"). Given that the λ-calculus and its extensions forms the basis for the majority of modern type theories, this work falls within the domain of Deliverable D4 ("Definition of a mathematical framework for modular reasoning about type theories and their extensions") and Research Coordination Objective 7 ("Develop a modular theory of type theories").

As mentioned, we expect that the algebraic understanding of the λ-calculus developed during the STSM will have applications also to more sophisticated calculi of interest in modern programming languages. Thus, this STSM is likely to be instrumental to developing an ongoing collaboration focused on applying category-theoretic techniques to the description of type theories and programming languages. As such, the STSM will contribute to achieving the following Capacity Building Objectives:

1. Bring together members of the different communities working on proofs in Europe.

3. Create an excellent and inclusive network of researchers in Europe with lasting collaboration beyond the lifetime of the Action.

5. Actively support young researchers, the under-represented gender, and teams from regions with less capacity.

7. Prepare competitive EU researchers for a fruitful career in an international environment through intensive use of Short Term Scientific Missions (STSM) and joint educational programs with industry.