Short-Term Scientific Mission Grant
- APPLICATION FORM[1] -

Action number: CA20111

Applicant name: Thomas Jan Mikhail

Details of the STSM

Title: The (∞,1)-category of ∞-groupoids in the spirit of Lawvere

Start and end date: 11.05.2025 - 24.05.2025

Detail of the cost in EUROS:

- Transport (upload screen capture): 116 EUR (867 DKK)

- Hotel/day (upload screen capture): 55 EUR/day (724 EUR total for 13 nights)

- Food/day: 15 EUR/day (210 EUR total for 14 days)

TOTAL: 1050 EUR

Goals of the STSM

Purpose and summary of the STSM.

Homotopy types have found their home in the language of higher category theory. From this perspective, homotopy types are understood as ∞-groupoids, and the step from classical category theory to higher category theory can then be understood as replacing sets with ∞-groupoids.

Lawvere spelled out an axiomatization of sets, called the Elementary Theory of the Category of Sets (ETCS), which describes the category of sets as a well-pointed topos with a natural number object. It is reasonable to expect ∞-category of ∞-groupoids to admit a description as the higher categorical version of Lawvere's ETCS. The purpose of this STSM is to make advancements towards such a description of the (∞,1)-category of ∞-groupoids within a type-theoretic framework and more generally to discuss foundational aspects of higher category theory.

In the recent years, homotopy type theory emerged as a synthetic language for the (∞,1)-category of ∞-groupoids, which has proven that the inductive nature of type theory is suitable for the description of higher categorical structures. It may be taken as evidence, that it is possible to spell out a foundational language in the framework and serves as a source of inspiration in the pursuit of the higher categorical analogue of Lawvere's ETCS.

(max.200 word)

Working Plan

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[1] This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via-e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

Description of the work to be carried out by the applicant.

Finster and Mimram introduced a type-theoretic description of an (∞,∞)-category called CaTT. This definition may be restricted to a definition $(\infty,1)$-categories which will serve as our starting point. As part of an effort to develop the theory of (∞,∞)-categories in the type theory CaTT, Mikhail proposed a definition for limits over finite computads. If restricted to $(\infty,1)$-categories, the situation might reduce considerably and allow for a simpler more workable definition for $(\infty,1)$-limits in $(\infty,1)$-categories. Once the notion of $(\infty,1)$-limit has been understood, the ideas used in its implementation may carry over to other basic categorical constructions, and in particularly those required for the description of the $(\infty,1)$-category of ∞-groupoids.

During the visit, Christian Sattler and I will work on the development of the higher analogue of Lawvere's ETCS. Concretely we work on the implementation of basic categorical structures in the restriction of CaTT to $(\infty,1)$-categories, with a special focus in the discussions put on $(\infty,1)$-limits. Thinking ahead, we will also discuss the prospects of how homotopy theory could be developed in a foundational setting based on the higher analogue of Lawvere's ETCS. The development of the higher analogue of Lawvere's ETCS will greatly benefit from such an interaction thanks to Christian Sattler's expertise in synthetic approaches to higher categories.

The Logic and Types group at the Chalmers University of Technology integrates other members interested in synthetic approaches to higher category theory. In addition to the discussions with Christian Sattler, the plan is to actively participate in the research group life and the local seminar and interact with the other members of the group. This visit would serve as an opportunity to meet other researchers with similar goals and increase networking.

(max.500 word)

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (https://europroofnet.github.io/objectives/) and deliverables (https://europroofnet.github.io/deliverables/).

Working groups to which this mission contributes: WG6 Type Theory

Type theory is the language and essential toolbox for computer verification. The homotopy interpretation of type theory is a key aspect, since, at the end of the day, verification involves the construction of a term in identity types, i.e. a homotopy. Now homotopy types, aka ∞-groupoids are just the invertible fragment of higher category. But the collection of all ∞-groupoid forms an $(\infty,1)$-category, because of which an interest in higher category theory becomes inevitable. In this sense, higher category indirectly already is a foundational tool for computer verification, and it is only a matter of time before higher category theory finds more direct applications. As an example, 'directed homotopy' (which is just another word for higher category theory) is already an important ingredient in concurrency and one may wonder whether higher categorical tools can be used for concurrency verification.

A research visit at the Chalmers University of Technology will help advance the development of the theory of $(\infty,1)$-categories in the type-theoretic framework of CaTT, forming the first steps towards a description of $(\infty,1)$-categories of ∞-groupoids. The type theory CaTT has already been implemented as a proof system. Therefore, any theory developed in CaTT has the potential of being implemented on top of the existing type system, providing us

with a computer verifiable theory for higher categories.

The visit will also serve as an opportunity to meet and interact with other researchers with similar interests to facilitate the exchange of ideas and promote networking.

(max.500 words)