

Short-Term Scientific Mission Grant - APPLICATION FORM¹ -

Action number: CA20111

Applicant name: Jacob Neumann

Details of the STSM

Title: Dinatural Semantics of Directed Type Theory

Start and end date: 07/05/2024 – 20/05/2024

Detail of the cost in EUROS:

- Transport (upload screen capture): €400

- Hotel/day (upload screen capture): €70

- Food/day: €59

TOTAL: €2206

Goals of the STSM

The purpose of this STSM is to develop a semantics of directed type theory that makes fundamental use of dinaturality. 'Directed type theory' refers to several proposed variants [7,15,13,12,1,5] of Martin-Löf type theory which replace the symmetric identity types with asymmetric "hom-types". Such type theories allow for synthetic reasoning about mathematical structures where direction and variance play a central role, such as categories, preorders, directed homotopy spaces, and computer processes/rewriting systems [4].

In several directed type theories of note (especially ones designed for reasoning about 1- and 2-categories, such as [1,7,12,13]), the asymmetry of hom-types is enforced by a modal typing discipline, which annotates terms as either "negative" (contravariant) or "positive" (covariant). However, frequently there are instances where the same term needs to be used both positively and negatively. [1] and [12] address this issue by restricting the theory to groupoid/neutral contexts and types, respectively. While this may suffice for some purposes, a more principled approach would be to include explicit syntax for handling divariance, drawing on the considerable category-theoretic literature [2,3,9,14] on dinatural transformations. We propose to do this by developing [8]'s type-theoretic syntax for dinaturality into a full semantics of directed type theory.

Working Plan

At the time of the STSM, the visitor, Jacob Neumann, will be a postdoctoral researcher at Reykjavik University. His forthcoming PhD thesis is on the semantics of directed type theory, expanding on the proposed theory of [1]. His PhD research also explored dinaturality, specifically the theory of strong dinatural transformations [11]. The host, Niccolò Veltri, is an assistant professor at the Tallinn University of Technology, with expertise on the topics of the project. Other members of the Logic and Semantics group and the Compositional Systems and Methods group at TUT are likely to contribute to

¹ This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via-e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

the mission of the STSM as well, including:

- Fosco Loregian, a researcher at TalTech and an expert on dinaturality and the coend calculus [9].
- Andrea Laretto, a PhD student under the supervision of Veltri and Loregian, and a coauthor of [8] along with Veltri and Loregian.
- Tarmo Uustalu, Neumann's supervisor at the time of the STSM and a frequent collaborator with the other participants. Uustalu is expected to be in Tallinn for the duration of the STSM.

Our research will focus on several questions:

1. What kinds of divariance ought to be permitted in a directed type theory? Divariant terms of a type? Types that depend divariantly on their context? Others? What is the operational behaviour of such constructs?
2. Can a dinaturality-based semantics of type theory be made to fit existing notions of 'model of type theory' (such as split comprehension categories, categories with families, categories with attributes, tribes, etc.), or is some modification of these notions necessary?
3. Does incorporating dinaturality into the semantics make these directed type theories better equipped at providing a synthetic language for category theory (especially the coend calculus)?
4. Does the non-composability of dinatural transformations pose a significant issue? Is it necessary to restrict to e.g. realisable dinatural transformations [2] or strong dinatural transformations [10,16,11]?
5. Can "dipresheaf models" (analogous to presheaf models [6]) of type theory be given?
6. How amenable are these constructions to computer formalisation?

Expected outputs and contribution to the Action MoU objectives and deliverables.

Working groups to which this mission contributes: **WG6**. This STSM primarily falls under the purview of Working Group 6: it is an investigation into possible extensions of standard type theories into their directed and dinatural variants. In particular, we aim to study the syntax and semantics of dinaturality-based directed type theories, and their potential use in proof assistants for reasoning about "directed" phenomena (such as categories).

This STSM primarily addresses the Action's **Deliverable 4**. We concern ourselves with both the semantics of type theory (how can dinaturality be incorporated into the semantics of directed type theory?) and the resulting syntax (how do divariant constructs in directed type theory actually work?).

This STSM also addresses several of EuroProofNet's objectives.

- **Research Coordination Objective 3:** there is a potential connection between directed type theory and the theory of concurrency [4,12], meaning that a better understanding of directed type theory potentially could improve our ability to reason about program correctness and verification.
- **Research Coordination Objective 7:** as mentioned, directed type theory is an important evolving branch of type theory. This project's contribution to directed type theory helps fill in our understanding of different possible type theories, and therefore make better general theories about type theories.
- **Capacity Building Objective 3:** we anticipate that this STSM will help further foster collaboration between the groups in Reykjavik and Tallinn, and provide a starting point for collaboration lasting well beyond the Action's end.

Capacity Building Objective 8: the participants have already begun to disseminate preprints outlining their approaches to directed type theory and dinaturality [1,8,11], and we anticipate that this STSM will

provide further material to be disseminated in preprints, talks, and published works.

References

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- [2] E. Bainbridge, P. Freyd, A. Scedrov, & P. Scott, "Functorial Polymorphism". 1990
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- [5] D. Gratzer, J. Weinburger, & U. Buchholtz, "Directed univalence in simplicial homotopy type theory". 2024
- [6] M. Hofmann, "Syntax and Semantics of Dependent Type Theory". 1995
- [7] D. Licata & R. Harper, "2-Dimensional Directed Dependent Type Theory". 2011
- [8] A. Laretto, F. Loregian, & N. Veltri, "Directed equality for (co)end calculus". 2024
- [9] F. Loregian, "Coend Calculus". 2021
- [10] P. Mulry, "Strong monads, algebras and fixed points". 1992
- [11] J. Neumann, "Paranatural Category Theory". 2023
- [12] P. North, "Towards a Directed Homotopy Type Theory". 2019
- [13] A. Nuyts, "Towards a directed homotopy type theory based on 4 kinds of variance". 2015
- [14] R. Paré & L. Román, "Dinatural Numbers". 1998
- [15] E. Riehl & M. Shulman, "A Type Theory for Synthetic ∞ -Categories". 2017
- [16] T. Uustalu, "Strong dinaturality and initial algebras". 2010