Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA20111

Grantee name: Jacob Neumann

Details of the STSM

Title: Second-Order Generalised Algebraic Theories for Modal and Substructural Type Theories

Start and end date: 24/06/2024 to 02/07/2024

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

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The aim of the STSM was to foster collaboration on the topic of second-order generalised algebraic theories, and explore possible extensions to modal and substructural type theories.

We spent several of the days of the STSM exploring directed type theory, and specifically the “minus” and “core” operations on the category of categories, as expressed in the categories with families structure on it (the category model). A minimal statement of the minus modality was found in terms of the context- substitution- and type-operations, and from this minimal presentation the “two-CwF” structure can be obtained. However, fundamental difficulties still prevent the translation of the two-CwF presentation back to the minimal endofunctor presentation, confirming that a proper extension of the SOGAT signature language is necessary in order to capture this modality.

We also focused on the theory of displayed models of generalised algebraic theories, their use in formalising models of type theory like the the setoid model and the groupoid model, and their connection to ongoing investigations into higher observational type theory. We initiated a formalisation of the groupoid model to explore the relative merits of displayed models versus more direct methods of formalisation.

Finally, we explored “crisp” modalities, to try and understand (1) the difference between crisp type

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

theory and 2-level type theory, (2) the extent to which crisp type theory makes closed terms and types accessible in arbitrary contexts, and (3) whether the standard model theory of crisp type theory is minimal. As a result, we were able to formulate a type theory which explicitly encodes whether variables depend on zero or one free variables.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

Grantee enters max 500 word summary here.

We made definite progress towards developing a modal extension of SOGATs, though we didn't obtain a concrete statement of such an extension. The STSM provided ample time for us to discuss and work out ideas, and deepen our understanding of various topics related to modal type theory and generalised algebraic theories. We will definitely continue collaborating on these topics, and work towards publishable results.

Some areas of planned continued collaboration:

- Further development of the Agda formalisation of the groupoid/category models of type theory started during the STSM
- Further discussions on directed and polarised type theory
- Development of higher observational type theory, particularly the simpler special case of groupoid type theory
- Continuing to explore questions related to crisp type theory, and further elaboration of the crisp/non-crisp type theory arrived at during the STSM

The STSM also proved very helpful for the development of the "Synthetic 1-Categories in Directed Type Theory" by Altenkirch and Neumann, which was submitted for POPL 2025 soon after the conclusion of the STSM.