

Report on the outcomes of a Short-Term Scientific Mission¹

Action number: CA20111

Grantee name: Jacob Neumann

Details of the STSM

Title: Dinatural Semantics of Directed Type Theory

Start and end date: 05/05/2025 to 17/05/2025

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

We were able to have a number of productive discussions about dinaturality and the semantics of type theory.

Early in the mission, I had the opportunity to present my system of directed type theory (and its semantics in the category model of type theory) for the regular Computer Science Theory Seminar (<https://niccoloveltri.github.io/tsem24/neumann.html>). In addition to prompting several insightful questions from members of the Logic & Semantics group, this presentation served as a starting point of conversation.

For instance, it prompted a discussion between myself, Andrea Laretto, and Niccolò Veltri concerning a potential counterexample to the category model semantics of directed type theory. In that system, if we have any operation K which (1) has the shape of the composition operator for directed paths, i.e. combining $f:\text{hom}(t,t')$ and $g:\text{hom}(t',t'')$ to $K(f,g):\text{hom}(t,t'')$; and (2) is propositionally unital in one argument, i.e. there is an identity between $K(f,\text{refl})$ and f ; then it can be proved internally (using the principle of directed path induction) that K must actually be the composition operator for directed paths, and in particular must be associative and unital in the other argument. The issue raised by Laretto and Veltri was: couldn't we introduce an operator K whose interpretation in the category model was specifically designed to *not* be associative or unital in both arguments, so the internal syntactic proof otherwise would lead to a contradiction? We eventually determined that, if such a K is expressible as a

¹This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

term in the theory (which it must be, in order for the internal proof to be possible), then its semantics includes a “naturality condition” which, when combined with the assumption of unitality in one argument, does imply semantically that K is the composition operator. Hence there is no contradiction.

I was also able to spend more time discussing the Laretto-Loregian-Veltri dinaturality type theory with Andrea Laretto, and better understand why the non-composability of dinatural transformations poses an obstacle for encapsulating the semantics of this theory as an abstract model notion. Based on these conversations, I began to sketch an abstract presentation of the predicates, entailments, etc. of this theory in the generalized algebraic theory style (à la categories with families) that I typically phrase semantics, but further work is required to fully develop this point.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

In part due to limitations of time (e.g. the STSM being shortened to 10 days instead of 2 weeks), I wasn’t able to make as much progress on incorporating dinaturality into the category model semantics of directed type theory as originally hoped. However, this STSM was successful in establishing a collaboration between myself and the researchers in Tallinn, that will hopefully lead to us addressing issues like this and perhaps reconciling our different approaches to directed type theory.

While more work is required to develop them further, I anticipate that the ideas developed during this STSM will be incorporated into future publications. Moreover, the discussions were particularly informative in the preparation of my PhD thesis—the above-mentioned conversation about alternative composition operators in the category model was specifically helpful in clarifying the nature of terms in the category model and how the variances of different terms involved in composition of directed paths is interpreted semantically.

Niccolò Veltri