Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA20111

Grantee name: Loïc Pujet

Details of the STSM

Title: Strictification of the syntax of type theory

Start and end date: 02/09/2024 to 06/09/2024

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

During the STSM in Budapest, Ambrus Kaposi and I worked together on the strictification of the syntax of type theory in intensional type theory, as described in the initial working plan. After some blackboard investigations of the different options, we picked the most promising one and we worked on an Agda formalisation of our strategy.

Our formalisation starts with an arbitrary category with families (i.e., an instance of a complicated record type involving fields and equations), and goes on to define a new category with families that is equivalent to contextual core of the first CwF, but for which all of the equations are definitionally true (except for one eta-equality). The definition is based on Pédrot's strict presheaf model [1].

We then went on to apply our construction to the initial CwF, which represents the syntax of type theory. Since the initial CwF is equivalent to its contextual core, our construction provides a fully strictified version of intrinsically typed syntax. We then did some experiments with canonicity proofs for this strictified syntax.

[1] Pierre-Marie Pédrot: Russian Constructivism in a Prefascist Theory. LICS 2020: 782-794

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications

1 This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

We were able to implement our strictification construction in Agda, for a very basic notion of CwF (the only type constructor is Pi-types). It seems clear that our method will scale to more complex notions of CwF, including universes, inductive types, and large elimination.

Our investigations with the canonicity proof have confirmed that using the strict syntax is a huge improvement over naive proofs: because all equations hold by definition, Agda is able to automate them and we do not have to deal with “transport hell”. Consequently, the canonicity proof becomes almost trivial. Once we extend our development to support large elimination, this will be the first fully formal gluing-style canonicity proof for dependent types [2, 3]. This is in direct contribution towards deliverable 15 of the Action MoU (Prototype implementation of the mathematical framework for modular reasoning about type theories and their extensions).

Our plans for the near future is to finish the proof, make it as modular as possible to permit re-use by the community, and publish our results. In addition to canonicity proofs, we plan to investigate *normalisation* proofs, which should also be made significantly easier by our strictification construction.

[2] Thorsten Altenkirch, Ambrus Kaposi: Normalisation by Evaluation for Dependent Types. FSCD 2016: 6:1-6:16

[3] Thierry Coquand: Canonicity and normalization for dependent type theory. Theor. Comput. Sci. 777: 184-191 (2019)