Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA20111

Grantee name: Mario Román

Details of the STSM

Title: A Type Theory for Exact and Continuous Bayesian Observations

Start and end date: 10/06/2024 to 14/06/2024

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

The visitor (Mario Román) and host (Elena Di Lavore) worked together during the five days of the STSM and some days before its start.

Most of the work revolved around the do-notation of copy-discard categories and effectful categories: we adapted our previous paper, "Evidential Decision Theory via Partial Markov Categories", to this notation, showing that nothing is lost by substituting string diagrams with a more traditional programming syntax. We considerably improved our manuscript "Partial Markov Categories", in preparation for submission to Logical Methods in Computer Science. We sketched how a rewriting mechanism translates exact observations in partial Markov categories to applications of a Bayesian inversion. This mechanism is particularly evident when employing a simple type theory such as do-notation for categories with copy-discard maps.

We finally developed a bunch of examples of Partial Markov Category syntax used to solve simple probabilistic programming riddles. Our inspiration was the recent work of Bart Jacobs on the two modes of Bayesian update (Pearl's method or Jeffrey's update rule). We used the do-notation already implemented in Haskell to get a simple draft implementation that answered the probabilistic riddles correctly (interestingly, employing Pearl's and Jeffrey's method depending on how the problem was stated and bringing our attention to the implicit assumptions on probabilistic programs).

Mario Román gave a seminar at the Computer Science Department of the University of Pisa on the type theory of premonoidal categories – which we take as inspiration for the type theory of partial

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

Markov categories. We acknowledge and thank the discussions with Filippo Bonchi, Fabio Gaducci, and Gabriele Tedeschi; we also thank discussions on Markov categories and traces with Jean-Simon Lemay and Zeinab Galal.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

The STSM contributed significantly to the planned goals. We still need to finish the journal submission, but we decided to include a section on the internal language of partial Markov categories and we developed it considerably. We still aim to publish this work in a relevant theoretical computer science journal (e.g., LMCS, TheoretiCS).

The collaboration will last beyond the lifetime of this action (Objective 3). The article we are preparing has as its main objective to ease formal verification in probabilistic domains (Objective 4). This collaboration has brought together researchers from different communities: Oxford, probabilistic programming, and Pisa, coalgebra, and monoidal categories in computer science (Objective 1).

The main outcome was the definition of a simple type theory for exact observations (D4), but also a sketched result of freeness that shows how it constructs the free partial Markov category on a signature. Our draft implementation (D15) tries to make techniques for program verification accessible to all stakeholders in probabilistic domains, where the lack of good formal notation is behind many apparent paradoxes. This research aligns with WG6.