Short-Term Scientific Mission Grant
- APPLICATION FORM[1] -

Action number: CA20111

Applicant name: Reuben Rowe

Details of the STSM

Title: Mapping the Landscape Between Proof Schemata and Cyclic Proofs

Start and end date: 06/04/2025 - 13/04/2025

Detail of the cost in EUROS:

As reference, you can use the daily allowances by country for ITCGs (https://europroofnet.github.io/itcg-daily-allowance/) and the associated Excel sheet (https://europroofnet.github.io/_pages/grant.xlsx).

[Please note that I have used the daily allowances quoted in the document available at http://www.cost.eu/daily_allowance that is linked to under the section "Reimbursement rules for STSMs" of the webpage https://europroofnet.github.io/grants/ rather than the daily allowances for ITCGs, which gives slightly lower allowances than for STSMs]

- Transport (upload screen capture): €397 (£330.22)

- Hotel/day (upload screen capture): €155 (£902.15 for a 7 night stay, see screen capture)

- Food/day: €50

TOTAL: €1832

Goals of the STSM

Purpose and summary of the STSM.

(max.200 word)

The use of non-wellfounded and cyclic derivations has emerged as a powerful proof-theoretic technique for capturing inductive and co-inductive reasoning principles across a range of logics (e.g. [1,2,3]).

On the other hand, proof schemata [4,5,6] have been developed as a representation of inductive proofs that are amenable to proof analysis, wherein information is extracted from proofs via normalisation, or cut-elimination.

So far, these two formalisms have been developed independently, and their interrelationships, and relative expressivity, have not yet been investigated.

This STSM aims to kickstart a new collaboration between Royal Holloway University of London (Reuben Rowe) and TU Wien (Stella Mahler, Alexander Leitsch, and Anela Lolić), to investigate precise relationships between the formalisms of cyclic proof and proof schemata, particularly in the context of arithmetic.

[1] This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

Specific objectives include:

- A syntactic translation from proof schemata into cyclic proofs.
- A description of the action of the cut-elimination by resolution (CERES) procedure at the level of cyclic proofs.
- Identification of subsystems of cyclic proofs corresponding to (extensions of) the proof schemata formalism, along with a constructive translation procedure from cyclic proofs to proof schemata.
- An action plan for implementing automatic translation between the two formalisms, possibly within the Cyclist theorem prover [7,8].

[1] Christoph Sprenger, Mads Dam. On the Structure of Inductive Reasoning: Circular and Tree-Shaped Proofs in the μ-Calculus. In Foundations of Software Science and Computation Structures. Springer Berlin Heidelberg. pp. 425–440.

[2] James Brotherston, Alex Simpson. Sequent Calculi for Induction and Infinite Descent. Journal of Logic and Computation 21(6), pp. 1177–1216.

[3] Alex Simpson. Cyclic Arithmetic Is Equivalent to Peano Arithmetic. In Foundations of Software Science and Computation Structures. Springer Berlin Heidelberg. pp. 283–300.

[4] Cvetan Dunchev, Alexander Leitsch, Mikheil Rukhaia, and Daniel Weller. Cut-elimination and Proof Schemata. In TbiLLC, volume 8984 of Lecture Notes in Computer Science, pages 117–136. Springer, 2013

[5] Alexander Leitsch, Nicolas Peltier, and Daniel Weller. CERES for First-order Schemata. J. Log. Comput., 27(7):1897–1954, 2017.

[6] David M. Cerna, Alexander Leitsch, and Anela Lolic. Schematic Refutations of Formula Schemata. J. Autom. Reason., 65(5):599–645, 2021.

[7] James Brotherston, Nikos Gorogiannis, and Rasmus L. Petersen. A Generic Cyclic Theorem Prover. In Programming Languages and Systems - 10th Asian Symposium (APLAS 2012). Lecture Notes in Computer Science, vol. 7705. Springer Berlin Heidelberg. pp. 350–367.

[8] https://github.com/cyclist-org/cyclist

Working Plan

Description of the work to be carried out by the applicant.

(max.500 word)

The STSM grant will fund a week-long trip for Reuben Rowe to visit Dr Anela Lolić, Prof. Alexander Leitsch, and his PhD student Stella Mahler, at TU Wien. Stella Mahler's thesis focuses on extending the proof schemata formalism, and Reuben Rowe has a track record of research in the area of cyclic and non-wellfounded proof theory, and is currently the main developer and maintainer of the Cyclist theorem prover tool. Reuben will be hosted by the Theory and Logic group at TU Wien, and will engage in daily extended discussions with Stella Mahler, Prof. Leitsch, and Dr Lolić.

Discussions on the first day will facilitate a knowledge exchange, aiming to establish a common working understanding of both proof schemata and cyclic proofs. Work over the following day will then

focus on formulating a translation from proof schemata to cyclic proofs. We expect this first task to be relatively straightforward, as the components of proof schemata are already sequent calculus derivations, that we should be able to combine in order to produce a cyclic proof object. Over the next two days, we will then investigate a reverse translation, taking cyclic proofs to proof schemata. Since cyclic proofs are at least as powerful as full systems of explicit induction (e.g. Peano arithmetic), for which Herbrand's theorem does not hold, it is likely we will in fact need to consider a restricted notion of cyclic proof. For this, we may be able to build upon existing results, in the vein of implicit complexity, linking restrictions on non-wellfounded proofs to specific complexity classes [9,10]. We will also discuss transferring the CERES (cut-elimination by resolution) technique over to the cyclic proof formalism, and compare it with existing notions of cut-elimination for non-wellfounded proofs (e.g. [11,12]), as well as recent work on extending Herbrand's theorem to the non-wellfounded setting [13]. We will use the final day to focus on drawing up a plan for implementing any techniques we have developed into the Cyclist automated theorem prover tool. Reuben Rowe has detailed knowledge about the implementation of Cyclist, and will help Stella Mahler develop the expertise needed to carry out development and extension of the Cyclist software.

[9] Gianluca Curzi, Anupam Das. Cyclic Implicit Complexity. In Proceedings of the 37th Annual ACM/IEEE Symposium on Logic in Computer Science. LICS '22. ACM. pp. 19:1–19:13.

[10] Gianluca Curzi, Anupam Das. Non-Uniform Complexity via Non-Wellfounded Proofs. In 31st EACSL Annual Conference on Computer Science Logic (CSL 2023). Leibniz International Proceedings in Informatics (LIPIcs), vol. 252. Schloss Dagstuhl – Leibniz-Zentrum für Informatik. pp. 16:1–16:18.

[11] Alexis Saurin. A Linear Perspective on Cut-Elimination for Non-wellfounded Sequent Calculi with Least and Greatest Fixed-Points. In Automated Reasoning with Analytic Tableaux and Related Methods. Springer Nature Switzerland. pp. 203–222.

[12] Borja Sierra-Miranda, Thomas Studer, and Lukas Zenger. Coalgebraic Proof Translations for Non-Wellfounded Proofs. In Advances in Modal Logic, AiML 2024, Prague, Czech Republic, August 19-23, 2024. College Publications. pp. 527-548.

[13] Bahareh Afshari, Sebastian Enqvist, and Graham E. Leigh. Herbrand Schemes for Cyclic Proofs. ILLC Preprint PP-2023-08, 2023.

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (https://europroofnet.github.io/objectives/) and deliverables (https://europroofnet.github.io/deliverables/).

Working groups to which this mission contributes:

(max.500 words)

The high-level expected output of the STSM is a concrete understanding of the relationship between the formalisms of cyclic proof and proof schemata, along with techniques allowing the transport of results from each formalism into the other. More immediately we will prepare the results of our research for submission to high quality, international research venues, such as peer-reviewed conferences and journals on computer science and logic.

The work in this STSM falls under the remit of WG1 (Tools on Proof System Interoperability) and WG2 (Automated Theorem Provers). It directly concerns the relationship between two important formalisms for (co)inductive reasoning, and aims to develop the theoretical underpinnings allowing translation between the two proof systems. Moreover, we aim to apply our theoretical results by extending the Cyclist automated theorem prover to implement the techniques that we develop for these two systems.

Our work will also contribute to Research Coordination Objectives 2 and 3 of the Action MoU; namely,

to "promote the output of detailed, checkable proofs from automated theorem provers", and "make techniques for program verification more effective and more accessible to all stakeholders". The literature on cyclic proof already demonstrates its utility for program verification, which often relies on reasoning effectively about inductively defined data, via its application to a wide range of logics (e.g. [14,15,16,17]). By mapping out the relationship between proof schemata and cyclic proofs, we will enable techniques for proof analysis, developed in the context of proof schemata, to be applied to the wide variety of program verification contexts for which cyclic proofs have been utilised. By implementing within the Cyclist theorem prover the theoretical techniques that we will develop, we will enhance the tool's ability to provide humans with detailed information implicit in the proofs that it generates.

Furthermore, this STSM will contribute to the following Capacity-building Objectives of the Action MoU.

(1) Bring together members of the different communities working on proofs in Europe.
(3) Create an excellent and inclusive network of researchers in Europe with lasting collaboration beyond the lifetime of the Action.
(5) Actively support young researchers, the under-represented gender, and teams from regions with less capacity.
(7) Prepare competitive EU researchers for a fruitful career in an international environment.

[14] James Brotherston, Richard Bornat, and Cristiano Calcagno. Cyclic Proofs of Program Termination in Separation Logic. In Proceedings of the 35th annual ACM SIGPLAN-SIGACT symposium on Principles of Programming Languages (POPL'08). ACM. pp. 101–112.

[15] Matteo Mio, Alex Simpson. A Proof System for Compositional Verification of Probabilistic Concurrent Processes. In Foundations of Software Science and Computation Structures - 16th International Conference, FOSSACS 2013. Lecture Notes in Computer Science, vol. 7794. Springer. pp. 161–176.

[16] Farzaneh Derakhshan, Frank Pfenning. Circular Proofs as Session-Typed Processes: A Local Validity Condition. Logical Methods in Computer Science 18(2), pp. 8:1–8:51.

[17] Gadi Tellez, James Brotherston. Automatically Verifying Temporal Properties of Pointer Programs with Cyclic Proof. In Automated Deduction – CADE 26. Lecture Notes in Computer Science, vol. 10395. Springer International Publishing. pp. 491–508.