EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA20111

Grantee name: Reuben Rowe

Details of the STSM

Title: Mapping the Landscape Between Proof Schemata and Cyclic Proofs

Start and end date: 06/04/2025 - 13/04/2025

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

Between Monday 7th April and Friday 11th April (inclusive), Reuben Rowe met daily with members of the Department at TU Wien. Meetings on Monday, Tuesday and Wednesday involved Anela Lolić, Alexander Leitsch, Stella Mahler, and Martin Riener. Meetings on Thursday involved Anela Lolić, Stella Mahler, and Martin Riener. Meetings on Friday involved Stella Mahler, Martin Riener, and David Cerna.

On Monday, the meetings centred around discussing connections between Proof Schemata and the cyclic proof formalisms, and developing a shared understanding of the two systems. Several examples from both formalism were discussed, and identified as starting points for the detailed comparison of both systems.

On Tuesday and Wednesday, the meetings focused on working through a particular example proof, namely the example from [1] that is used as a counterexample to demonstrate the failure of cut elimination in the cyclic proof system for first order logic with inductive definitions. The work carried out involved reformulating this example in the proof schemata formalism, and beginning to work out the operation of the CERES cut elimination method on this example. Stella Mahler and Martin Riener began formalising this example as an input to the GAPT software system. The discussion also involved how to translate from proof schema to cyclic arithmetic proofs: it appears straightforward to combine the two

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

components of a proof schema into the zero and successor sub-derivations of a numeric case split rule, and convert recursive proof links into cyclic backlinks.

On Wednesday, Reuben Rowe presented on the topic of algorithms for deciding the Infinite Descent property in the Departmental Logic Seminar.

On Thursday, the discussion centred on a further example from the cyclic proof literature, namely the 2-hyrda example from [2,3] used to show the inequivalence of cyclic and explicit induction proofs in first order logic with inductive definitions. Reuben Rowe described the example in detail so that the team from TU Wien could understand it fully. Reuben Rowe also explained how this example admits an explicit induction proof, which could be formulated as a proof schema, and to which the CERES method could be applied.

On Friday, Martin Riener demonstrated some details of the GAPT software system to Reuben Rowe. Stella Mahler and David Cerna also discussed some details of the schematic unification problem, which is a central aspect of the CERES method for proof schema. An example from the proof schema literature, namely a statement of the infinite pigeonhole principle was discussed, and identified as a useful example to formalise in the setting of cyclic proofs. It was observed that a related example is analysed in [4], in the context of cyclic proofs, and agreed that this should be further studied.

[1] Yukihiro Oda, James Brotherston, Makoto Tatsuta: The failure of cut-elimination in cyclic proof for first-order logic with inductive definitions, Journal of Logic and Computation, Volume 35, Issue 2, March 2025, exad068, https://doi.org/10.1093/logcom/exad068
[2] Stefano Berardi, Makoto Tatsuta: Classical System of Martin-Löf's Inductive Definitions Is Not Equivalent to Cyclic Proof System. FoSSaCS 2017: 301-317
[3] Stefano Berardi, Makoto Tatsuta: Classical System of Martin-Löf's Inductive Definitions is not Equivalent to Cyclic Proofs. Log. Methods Comput. Sci. 15(3) (2019)
[4] Bahareh Afshari, Sebastian Enqvist, and Graham E. Leigh. Herbrand Schemes for Cyclic Proofs. ILLC Preprint PP-2023-08, 2023.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

The STSM has facilitated extremely useful discussions and knowledge exchange on the relationships between proof schemata and cyclic proof. Many key examples were discussed, which will serve as the basis of case studies for linking to two formalisms. A clear translation from the proof schemata to cyclic arithmetic proofs was identified. The STSM has achieved its primary goal of kickstarting a new collaboration to link proof schemata and cyclic proof, and to study common techniques for cut elimination in these systems. It has also led to a concrete action plan for further collaboration to continue addressing the specific objectives of describing the action of the CERES procedure in the cyclic proof formalism, and implementing automatic translations between the formalisms.

All researchers involved in the STSM agreed that the week had been extremely fruitful, and contributed to the intended capacity-building objectives of the COST action MoU. Namely, to bring together our two communities of research, creating lasting collaboration beyond the lifetime of the Action, and actively supporting young researchers.

A number of follow up objectives were identified in the meetings on Thursday 10th April.

1. Fully work through the "left and right addition" example from [1].
    - Finish the full construction of the proof schema discussed during the meetings.
    - Formalise the proof scheme in GAPT
    - Analyse the proof scheme using the CERES method.

2. Study further the 2-hydra example from [2,3]: formalise the explicit induction proof as a schema and determine what the action of the schematic CERES method on this schema is? In particular, the key aspect to investigate is whether or not explicit reasoning about the ordering relation on natural numbers is eliminated (viz. it does not appear in the cyclic proof).

3. Outline in detail translations between proof schemata and cyclic arithmetic proofs. This includes the straightforward translation from proof schemata to cyclic proofs described above.

4. Investigate whether the cyclic proof of totality of the Ackermann-Péter function (cf. [5]) can be represented as a proof schema.

5. Study the outcome of performing the non-schematic CERES method on cyclic proof derivations in which backlinks are discarded. The key questions here are whether refutations of the resulting characteristic formulae exist in general, and whether the CERES method preserves the buds of the derivation, in which case does reintroducing the backlinks from the original proof preserve the Infinite Descent property?

6. A comparison of the Herbrand schemes obtained by the schematic CERES method with the construction of Herbrand systems given in [4].

7. A comparison of the schematic CERES method with the (infinitary) reductive cut elimination strategy described in [6].

8. [5] Alex Simpson. Cyclic Arithmetic Is Equivalent to Peano Arithmetic. In Foundations of Software Science and Computation Structures. Springer Berlin Heidelberg. pp. 283–300. [6] Graham E. Leigh, Bahareh Afshari. A metapredicative study of μ-arithmetics. August 2024. GUPEA: https://hdl.handle.net/2077/84780.

Anela Lolić
Senior PostDoc, Institute of Logic and Computation, TU Wien
Host of the STSM