Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA20111

Grantee name: Anton Setzer

Details of the STSM

Title: Formalisation of Meaning Explanations in Agda

Start and end date: 23/08/2022 to 06/09/2022

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

All formalisations in type theory in the following have been carried out in the theorem prover Agda.

1) A substantial part of the formalisation of system T in type theory has been carried out, some theorems are still postulates.

2) A substantial part of the meaning explanations for Pi types have been formalised. In order to accelerate the progress, we switched to formalising combinatorial algebra instead of a version based on the lambda calculus in type theory. This allowed us to formalise explicit mathematics, in which the original version of the extended predicative Mahlo universe was formalised.

3) Standard data types from explicit mathematics have been formalised in type theory.

4) We have formalised universes in explicit mathematics in type theory.

5) An extension to full inductive-recursive definitions is still pending.

6) We have formalised the extended predicative Mahlo Universe based on explicit mathematics in type theory.

1 This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

** This included a variation of the original Mahlo universe, namely an external extended predicative Mahlo universe.*

** Meaning explanations for the original extended predicative Mahlo universe turned out to be difficult to justify. Therefore, we defined a version where the subuniverses are first built by an inductive definition (which doesn't refer directly to the Mahlo universe) and only later added to the Mahlo universe.*

** The formalisation has been completed in Agda. Moreover, a corresponding induction principle, stating that the extended predicative Mahlo universe is the minimal one, has been developed.*

** The resulting type theory is to our knowledge the currently strongest fully predicatively justifiable extension of Martin-Löf Type Theory.*

** This part of the research has been described in a paper (27 pages). We intend to submit it to the post-proceedings of the conference 'Celebrating 90 Years of Gödel's Incompleteness Theorems' which was held in 2021.*

7) Principles for induction over the new universes in type theory and explicit mathematics have been developed.

8) The more speculative part regarding Cubical Type Theory turned out to be more complex than expected. Therefore, we decided to first finish the above formalisations before dealing with cubical type theory.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

Achievements:

We have developed a library for formalising meaning explanations in the theorem prover Agda.

We have developed an embedding of the extended predicative Mahlo universe in Agda.

We have written a 27 page draft paper documenting the embedding of the extended predicative Mahlo Universe in Agda. This is to be submitted to the post-proceedings of the conference 'Celebrating 90 Years of Gödel's Incompleteness Theorems' which was held in 2021.

The paper will be accompanied by a git repository with the Agda code.

Followup activities:

We are working on finalising the paper and submitting it to the Gödel conference mentioned above. We will create an accompanying git repository containing the full Agda code.

We are planning to

** Prove that all theorems in explicit mathematics are provable in type theory and carry out this proof in Agda.*

** Develop a model of the resulting type theory which embeds the Mahlo universe in an extension of Kripke-Platek set theory. This will allow us to determine an upper bound for its proof theoretic strength.*

** Adapt the approach to using the lambda calculus instead of combinatory logic.*

The resulting work is planned to be published in two longer journal papers. In the first we plan to prove conservativity of the embedding of explicit mathematics in Agda. This will also be accompanied by a git repository containing the full Agda code.